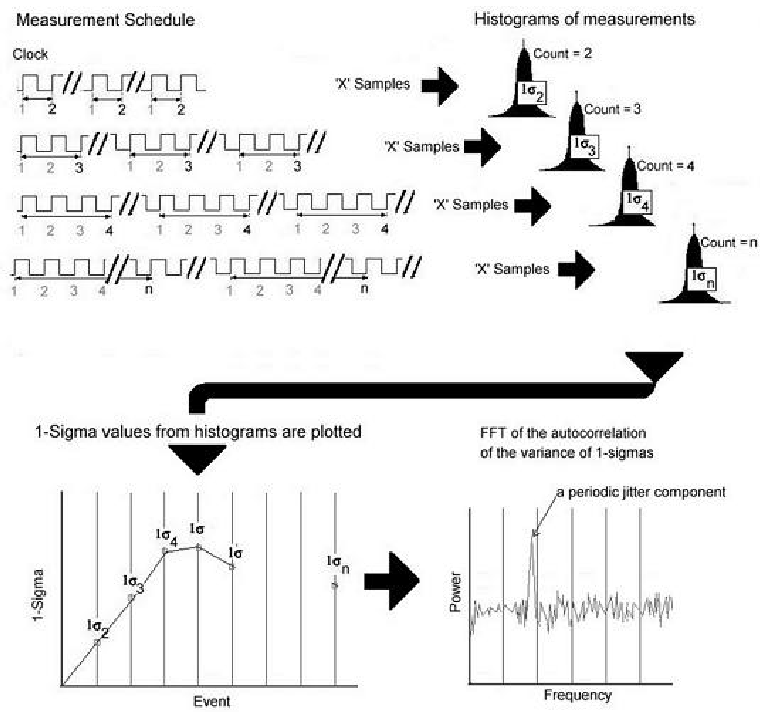
Jitter Definitions

Accumulated time analysis

A proprietary technique for calculating the jitter spectrum of a data stream via direct measurements of the autocorrelation function of jitter. The process is as follows. The edge-to-edge timing of a one unit interval (UI) relationship is measured repeatedly and asynchronously. A histogram of the measurements is constructed and the rms value is determined and is denoted by s_1. Then, the edge-to-edge timing of a two UI relationship is measured similarly, yielding an rms value of s_2. The process is repeated for up to *n* UI. The series of rms values (s_1 to s_n) is plotted as a function of time (in units of UI). Each s represents the amount of jitter accumulated over a certain span of time. It has been shown that the set of variance (s^2) values as a function of time is equivalent to the autocorrelation function of the jitter. Thus, by the Blackman-Tukey method, a Fourier transform of the autocorrelation function will yield the power spectral density (PSD) of the jitter. The process is illustrated below.



The highest frequency of the jitter spectrum is determined by the Nyquist frequency of the sampling interval. For accumulation in steps of one UI, the corresponding Nyquist frequency is one-half of the carrier frequency. The lowest frequency of the jitter spectrum is similarly determined by the longest span of UIs over which jitter is allowed to accumulate.

Autocorrelation function

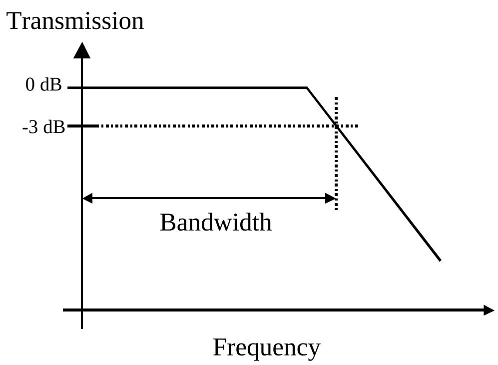
The correlation between the values of a random process at two different times. For a stationary random process, it is given by,

$$R_{xx}(t) = \int_{-\infty}^{\infty} x(t)x(t+t)dt \,,$$

where $x(t)$ is a sampling function of the random process, t and t are times, and T is the total time span of the sampling. A Fourier transform of the autocorrelation function will give the power spectral density of the $x(t)$ (see Blackman-Tukey method).

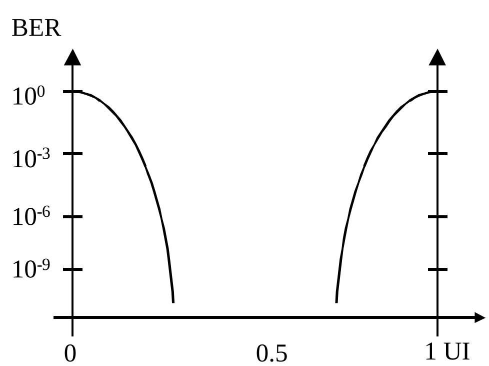
Bandwidth

The characteristic frequency range relevant to a particular application. Usually defined by the -3 dB point(s). For example, the bandwidth of a low pass filter is the range from 0 Hz up to the -3 dB cut-off frequency, as illustrated below.



Bathtub curve

A plot of the bit error ratio (BER) as a function of the unit interval (UI). In terms of the eye diagram, the bathtub curve indicates the eye opening for a particular BER and reference voltage. The illustration shows a typical bathtub curve. The probability of an erroneous bit is high near the ideal transition edge at 0 UI and 1 UI,. The probability of error decreases towards the center of the UI.



Bit Error Ratio (BER)

In a communications system, it is the number of erroneous bits received (or transmitted) divided by the total number of bits received (or transmitted).

Blackman-Tukey method

A method for determining the power spectral density (PSD) by calculating the Fourier transform of the autocorrelation function,

$$S(w) = \int_{-\infty}^{+\infty} R(t) e^{-jwt} dt$$
$$R_{xx}(t) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} x(t)x(t+t)dt ,$$

where $S(w)$ is the PSD, w is angular frequency, t and t are times, $x(t)$ is the sampling function, and T is the total time span of the sampling. Alternatively, by the Wiener-Khinchine theorem, it can be shown that

$$S(w) = \lim_{T \to \infty} \frac{\left| \int_{-\infty}^{+\infty} x(t) e^{jwt} dt \right|^2}{T} ,$$

which is the modulus square of the Fourier transform divided by the total time span.

Convolution

Mathematical process defined by the integral

$$y(t) = h(t) \otimes x(t) = \int_{-\infty}^{+\infty} h(t)x(t-t)dt .$$

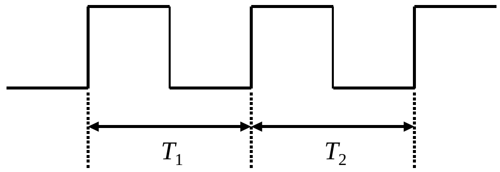
In other words, the input $x(t)$ is "folded" or "smeared" with the weighing function $h(t)$ to produce output $y(t)$. Equivalently, the convolution integral can be expressed as

$$y(t) = \int_{-\infty}^{+\infty} Y(w) e^{-jwt} dw ,$$
$$Y(w) = H(w)X(w)$$

which is the inverse Fourier transform of the products $H(w)$ and $X(w)$, where $H(w)$ and $X(w)$ are Fourier transforms of the functions $h(t)$ and $x(t)$.

Cycle-to-cycle jitter

Also known as "adjacent cycle jitter". High frequency timing deviations determined by the difference between the periods of adjacent clock cycles. In the illustration below, cycle-to-cycle jitter would be T_1-T_2.



Data dependent jitter (DDJ)

A component of deterministic jitter (DJ). Also known as Pattern Dependent Jitter. A source of jitter whose properties are dependent on the data being transmitted, e.g. duty cycle distortion (DCD) and inter-symbol interference (ISI). Clock signals and clock-like signals (10101010…, 110011001100…, etc.) have no variation in pattern. Thus they do not suffer from ISI.

Delta function

Also known as “Dirac delta function”. A discontinuous function that has the following properties:

$$d(x-x_0)=0, \qquad \text{for } x \neq x_0,$$

$$\int_{-\infty}^{+\infty} d(x-x_0)dx = 1,$$

$$\int_{-\infty}^{+\infty} f(x)d(x-x_0)dx = f(x_0).$$

In actuality, $d(x\text{-}x_0)$ is not a function at all, since it is undefined (infinite) at $x = x_0$. Analytically, the delta function can be expressed as

$$d(w-w_0)=\frac{1}{2p}\int_{-\infty}^{+\infty} e^{i(w-w_0)t}dt\,,$$

which is the Fourier transform of the sinusoidal function e^{-iwt}. Thus, for a purely sinusoidal function, the spectrum is an infinitely sharp spike at the oscillating frequency.

Deterministic jitter (DJ)

A component of total jitter (TJ). Also known as “systematic jitter.” Jitter with non-Gaussian probability density function. Deterministic jitter is always bounded in amplitude and has specific causes. Four kinds of deterministic jitter are identified: duty cycle distortion (DCD), inter-symbol interference (ISI), periodic or sinusoidal (PJ), and bounded but uncorrelated to the data (BUJ). DJ is characterized by its bounded, peak-to-peak value as well as its probability density function (PDF).

Double delta model

A mathematical description of the simplest case of non-zero deterministic jitter (DJ). It assumes that the DJ probability density function (PDF) is comprised of two delta functions.

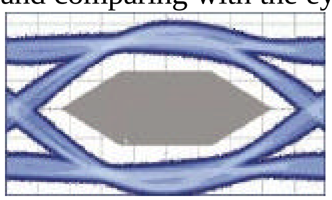
This DJ distribution corresponds to a perfect square wave modulation with no other sources of jitter. The common expression for total jitter (TJ) at 10^{-12} BER given by TJ = DJ + 14 RJ is a result of this overly simplified model. In general DJ distributions are not represented by delta functions. Therefore, under realistic circumstances, this expression is only an approximation.

Duty cycle distortion (DCD)

A component of deterministic jitter (DJ) and data dependent jitter (DDJ). DCD is the difference in the mean pulse width of a '1' pulse compared to the mean pulse width of a '0' pulse in a clock-like (10101…) bit sequence.

Eye Diagram

A simplistic and crude visualization of signal integrity, usually performed with a sampling oscilloscope. An eye diagram consists of multiple traces of data bits triggered by a bit clock. The traces are superimposed in persistence mode showing the envelope of amplitude and timing fluctuations. The region in the center that is devoid of any traces typically resemble an eye, thus the descriptive name "eye diagram." Signal integrity and jitter tolerance may be specified by defining an eye mask and comparing with the eye opening.



Fourier transform

An integral transform pair defined by

$$f(t) = \frac{1}{2p}\int_{-\infty}^{\infty} F(w)e^{jwt}dw$$
$$F(w) = \int_{-\infty}^{\infty} f(t)e^{-jwt}dt ,$$

where the two equations are commonly referred to as the forward and inverse transforms, respectively. The Fourier transform is used in spectral analysis of waveforms. For example, a phenomenon in the time domain $f(t)$ can be transformed into the frequency domain $F(w)$ to facilitate the study of its spectral composition.

FFT (fast Fourier transform) or DFT (discrete Fourier transform)

The Fourier transform is only valid for a continuous set of values. However, for discrete data, the discrete time and frequency values are given by

$$t_k = \frac{kT}{2N} \text{ and } w_p = \frac{2pp}{T} ,$$

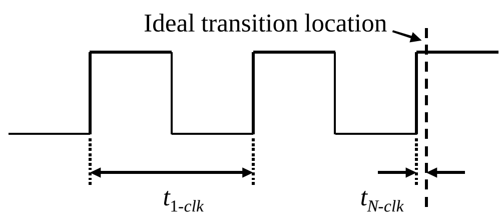
where $2N$ is the number of discrete values, k = 0,1,2,…,$2N$-1 and p = 0,1,2,…,$2N$-1. The DFT is given by

$$f(t_k) = \sum_{p=0}^{2N-1} F(w_p) e^{-jw_p t_k}$$
$$F(w) = \frac{1}{2N} \sum_{k=0}^{2N-1} f(t_k) e^{jw_p t_k} .$$

The FFT is a particular numerical method for performing a DFT which drastically reduces the number of numerical operations needed as long as N is a power of 2. For a DFT, the number of operations scale as N^2. For an FFT, the number of operations scale as $(N/2)\log_2(N)$. Thus, for N = 1024, the FFT reduces computation time by a factor of over 200.

FFT 1-clk and FFT N-clk

Spectral views available in the visualization of jitter spectra. The difference lies in the measurement reference. For FFT N-clk, jitter is measured relative to the ideal transition location. For FFT 1-clk, jitter is measured relative to a previous transition edge. For example, consider a clock with frequency f_c and ideal period T_0. Time measurements relative to different reference points are illustrated below.



It can be shown that the relationship between $t_{1\text{-}clk}$ and $t_{N\text{-}clk}$ is, to first order,

$$t_{1-clk} \approx \frac{dt_{N-clk}}{dt} T_0 + T_0 .$$

In the frequency domain, we get

$$T_{N-clk}(f) = \frac{T_{1-clk}(f)}{2p} \frac{f_c}{f} ,$$

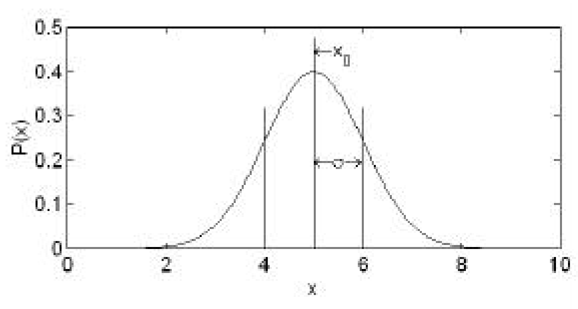
where $T_{N\text{-}clk}$ and $T_{1\text{-}clk}$ are the spectra for 1-clk and N-clk measurements, respectively. Thus, the FFT 1-clk view and the FFT N-clk view differ by $1/f$ despite having difference measurement references.

Gaussian distribution

Also known as the normal distribution and is defined by

$$p(x) = \frac{1}{\sqrt{2p}\, s} e^{-\frac{(x-\bar{x})^2}{2s^2}} ,$$

where $\bar{x}$ and s are the mean and standard deviation of the distribution, respectively. The familiar bell-shaped Gaussian probability density function (PDF) is shown below.



The distribution has a symmetrical peak centered at $\bar{x}$. The tails extend to $\pm\infty$ and the area under the curve is equal to unity. Approximately 68% of all events fall within $\pm s$, 95% fall within $\pm 2s$, and 99% fall within $\pm 3s$. By the central limit theorem, radom processes approach a Gaussian distribution for a sufficiently large number of statistically independent samples. Random jitter (RJ) obeys the Gaussian distribution.

Inter-symbol interference (ISI)

A component of deterministic jitter (DJ) and data dependent jitter (DDJ). ISI is a timing error correlated to the pattern of previous bits. One of the sources for this error is bandwidth limiting effects. Different data patterns have different frequency components and will disperse differently through a bandwidth-limited medium, causing timing errors. Another effect is exponential rising and falling edges. For example, a long run of ones (011110…) will result in a higher voltage amplitude, which will delay the falling edge. Similarly, a sequence of alternating ones and zeros (101010…) will have a lower voltage amplitude, which results in an earlier falling edge.

Jitter Output

The quantity of jitter at a specific physical position in a device or system under test.

Jitter tolerance (receiver)

The ability of a CDR circuit to recover an incoming data stream correctly despite jitter. It is characterized by the amount of jitter required to produce a specified bit error rate. The tolerance depends on the frequency content of the jitter.

Jitter transfer (transmitter/medium/receiver)

The ratio between the jitter output and jitter input for a component, device, or system often expressed in dB. A negative dB jitter transfer indicates the element attenuated jitter. A positive dB jitter transfer indicates the element amplified jitter. A zero dB jitter transfer indicates the element had no effect on jitter. The ratio should be applied separately to deterministic jitter components and Gaussian (random) jitter components.

Mean

Also known as the average value. Defined analytically as

$$\bar{x} = \int_{-\infty}^{+\infty} xp(x)dx$$

where x is a member of the population and $p(x)$ is the probability density function of the population. For a sample population, the expression becomes

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N},$$

where x_i is a member of the sampling and N is the total number of samples.

Peak-to-peak jitter

A description of the amplitude of jitter using only the extrema. This is usually applied to bounded forms of jitter distributions (e.g., DJ, PJ, DCD, and ISI).

Periodic Jitter (PJ)

A component of DJ. Also known as sinusoidal jitter. PJ is a timing error whose amplitude is oscillates sinusoidally with time. PJ can also consist of multiple sinusoids. For example, PJ could be the result of unwanted modulation, such as electromagnetic interference (EMI). PJ is quantified as a pk-pk number, specified with a frequency and magnitude. The behavior of PJ on a clock or data signal is identical to frequency or phase modulation.

Power spectral density (PSD)

A spectrum of the power density, usually denoted by the symbol $S(f)$. By definition, PSD is found by the Blackman-Tukey method. The PSD measures the distribution of noise power with frequency. The integral over all frequencies of $S(f)$ is the total power in the noise.

Probability Density Function (PDF)

In the case of jitter, PDF is the probability per unit time as a function of time. The total jitter PDF over all time is normalized, i.e.

$$\int_{-\infty}^{\infty} p(x)dx = 1.$$

It is important to note that the values of $p(x)$ are not probabilities. The probability of an event lying in the interval between a and b is given by

$$\int_{a}^{b} p(x)dx .$$

Random Jitter (RJ)

A component of total jitter (TJ). RJ is due to random fluctuations and noise sources. Examples include flick noise, shot noise, thermal noise, etc. By the central limit theorem, a large number of statistically independent events will approach a Gaussian distribution. Long term ($>10^{12}$ bits) signal integrity is strongly dependent on the magnitude of RJ.

Standard deviation and root-mean-square (RMS) deviation

A measurement of the width of the distribution of the displacement about the mean. Analytically, both are defined by

$$s = \sqrt{\int_{-\infty}^{+\infty} (x - \bar{x})^2 \, p(x) dx}$$

where x is a member of the population, $p(x)$ is the probability density function of the population, and $\bar{x}$ is the mean of the population. For a sample population, the RMS deviation is defined by

$$s_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} \, ,$$

where x_i is a member of the sampling and N is the total number of samples. Similarly, for a sample population, the standard deviation is defined by

$$s_{St.Dev.} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \, .$$

The difference in the pre-factor lies in the concept of (N-1) degrees of freedom for N independent samples. For N , s_{RMS} and $s_{St.Dev.}$ are equal.

Stochastic (random) process

Phenomena or process that are random as a function of time. For example the thermal noise voltages of a resistor in an electronic device. These processes generally have a Gaussian distribution.

TailFit™

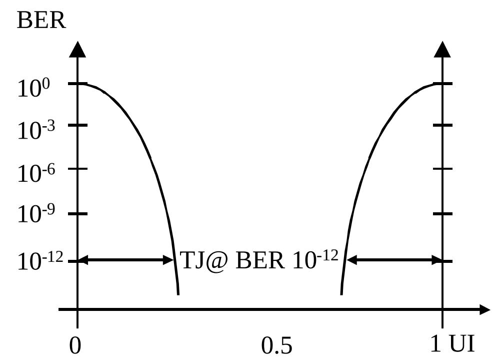
A proprietary algorithm developed by Wavecrest Corporation to separate random jitter (RJ) and deterministic jitter (DJ) from the total jitter (TJ) histogram. It is known that RJ has an unbounded Gaussian distribution and that TJ histograms always have Gaussian tails due to the bounded nature of DJ. Therefore, a least-squares fit of the tail regions to Gaussian functions will yield the rms value of RJ. The mean locations of the Gaussians for the left and right tails is indicative of the peak-to-peak values of the DJ distribution.

Timing Jitter

In binary data communications, jitter is defined as any deviation in the timing of a data transition edge from the ideal timing location.

Total Jitter (TJ)

Total jitter is function of bit error ratio (BER) and is, in principle, unbounded. Therefore, TJ is only meaningful when specified at a certain BER. In terms of a BER bathtub curve, TJ is the amount of eye closure at a particular BER. TJ consists of two main components: deterministic jitter (DJ) and random jitter (RJ). The TJ distribution is the convolution of the DJ and RJ distribution.



Unit interval (UI)

A unit of time corresponding to the transmission/reception of one bit of data. The reciprocal of UI is Baud in units of (bits/sec).

Variance and mean square error

The mean squared difference between a member of the population and the mean value of the population. Analytically, the variance is simply the square of the standard deviation,

$$s^2 = \int_{-\infty}^{+\infty} (x - \bar{x})^2 \, p(x) dx \,,$$

where x is a member of the population, $p(x)$ is the probability density function of the population, and $\bar{x}$ is the mean of the population. For a sample population, the variance is defined as

$$s^2_{St.Dev.} = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 \,,$$

where x_i is a member of the sampling and N is the total number of samples. Similarly, for a sample population, the mean square error is defined by

$$s^2_{RMS} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 \,.$$

The differences, again, are attributed to the number of degrees of freedom. For N , both variance and mean square error are equal.

Wander

Also known as “drift.” Long-term random variations in the signal. Typical time scale for such variations is on the order of ~10 Hz.