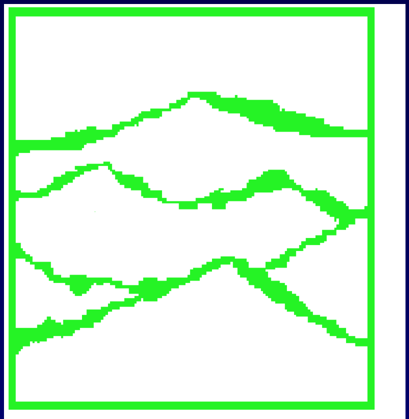
Designcon 2002 Presentation



A New Method For Simultaneously Measuring And Analyzing PLL Transfer Function And Noise Processes

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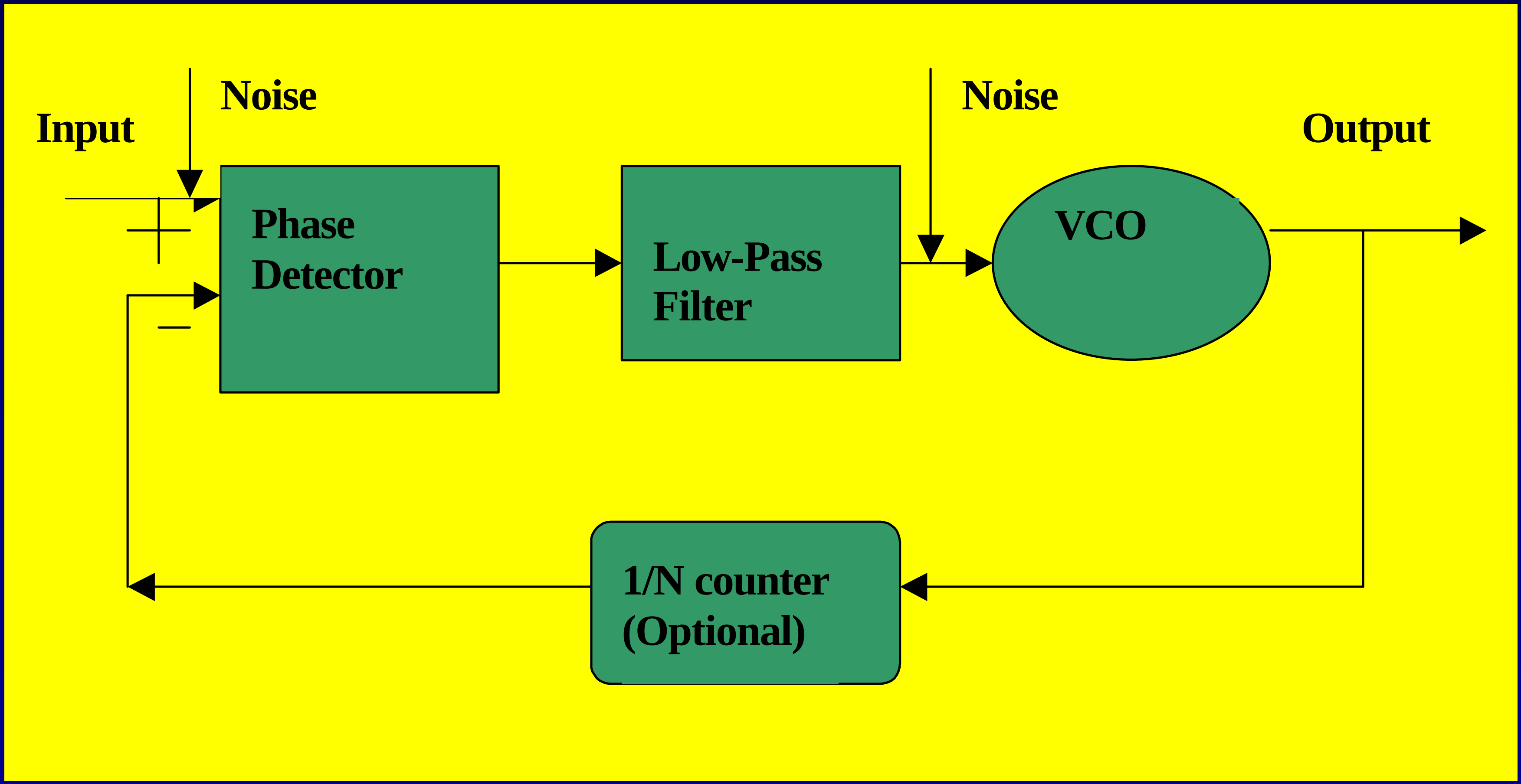
Outline

- Introduction
- Phase Locked-Loop (PLL) and Noise Processes
- Variance and Power Spectrum Density (PSD)
- Application of Variance and PSD in PLL analysis
- Conclusion

(Patents for the methodology is pending)

Phase-Locked Loop



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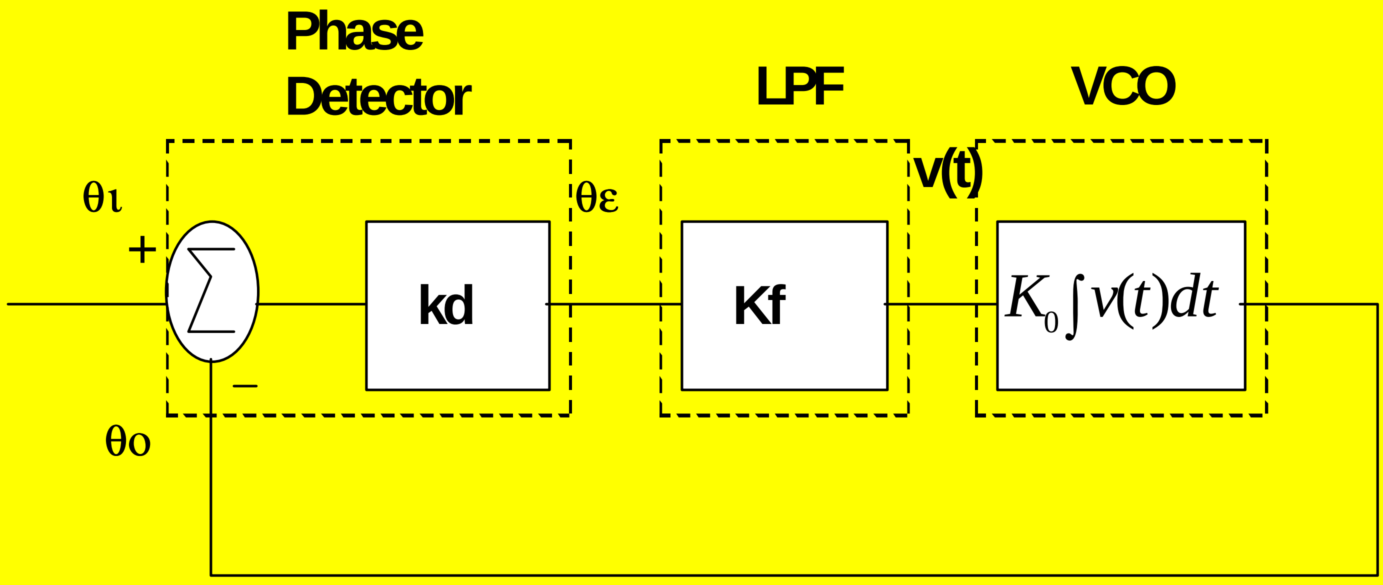
PLL Applications

- Tracking
- Frequency multiplication / division
- Synchronization
- Demodulation
- Computer / microprocessor
- Clock generation
- Clock recovery

PLL Model

I.) Time-Domain Approach



PLL Model

I.) Time-Domain Solutions

$$q_e = e^{-Kt}\left(\int e^{Kt} q_i(t)\,dt + c\right)$$

$K = K_d\, K_f\, K_{o,}$ is the loop gain,

If θ_i = constant , then $\theta_e \rightarrow 0$ when $t \rightarrow \infty$;

If $\theta_i = \omega t$, then, $\theta_e \rightarrow$ constant when $t \rightarrow \infty$.

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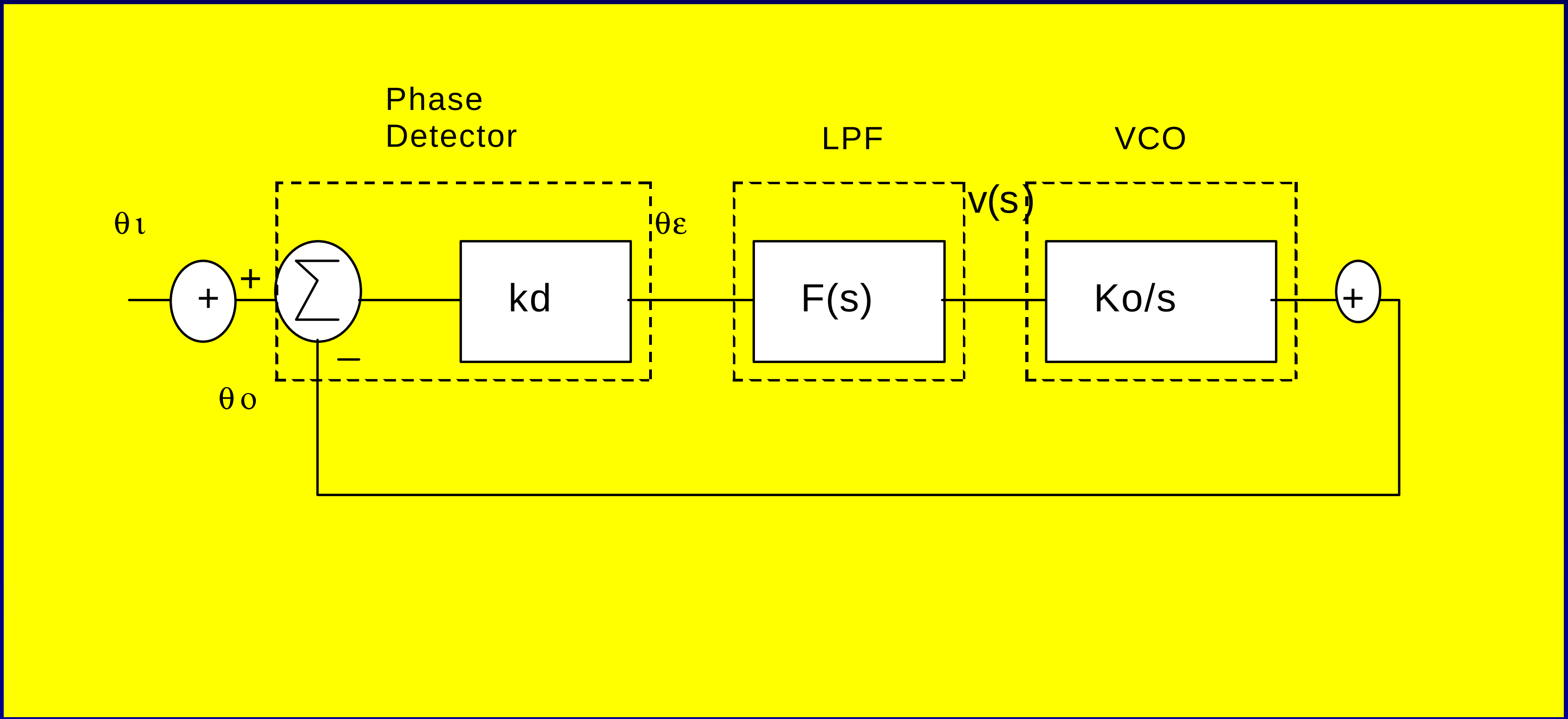
PLL Model

II.) Frequency-Domain Approach



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PLL Model

II.) Frequency-Domain Solution

- System transfer function

$$H_o = \frac{q_0(s)}{q_i(s)} = \frac{K_d K_o F(s)}{s + K_d K_o F(s)}$$

- Error transfer function

$$H_e(s) = \frac{q_e(s)}{q_i(s)} = \frac{K_d s}{s + K_d K_o F(s)}$$

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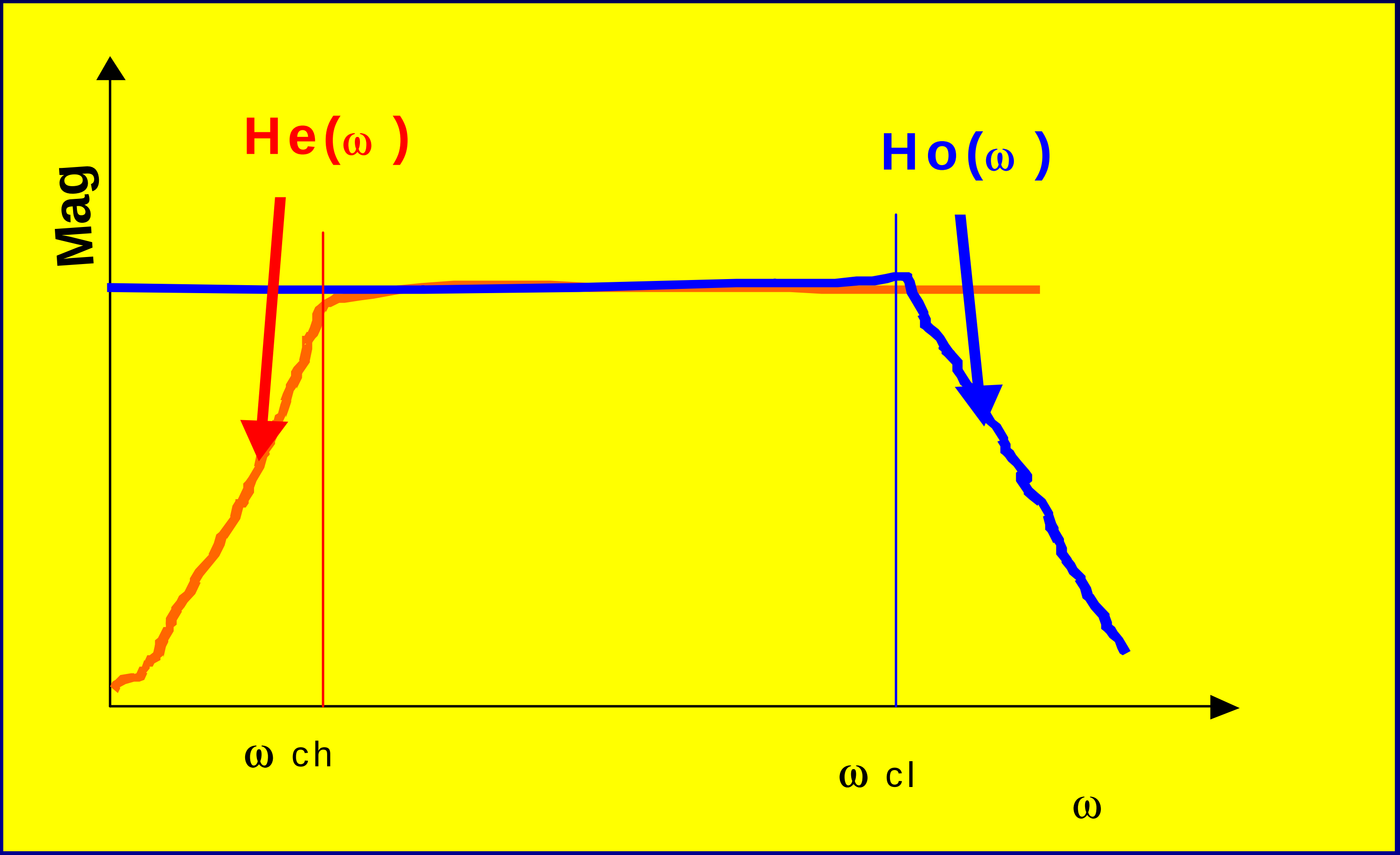
PLL Transfer Functions



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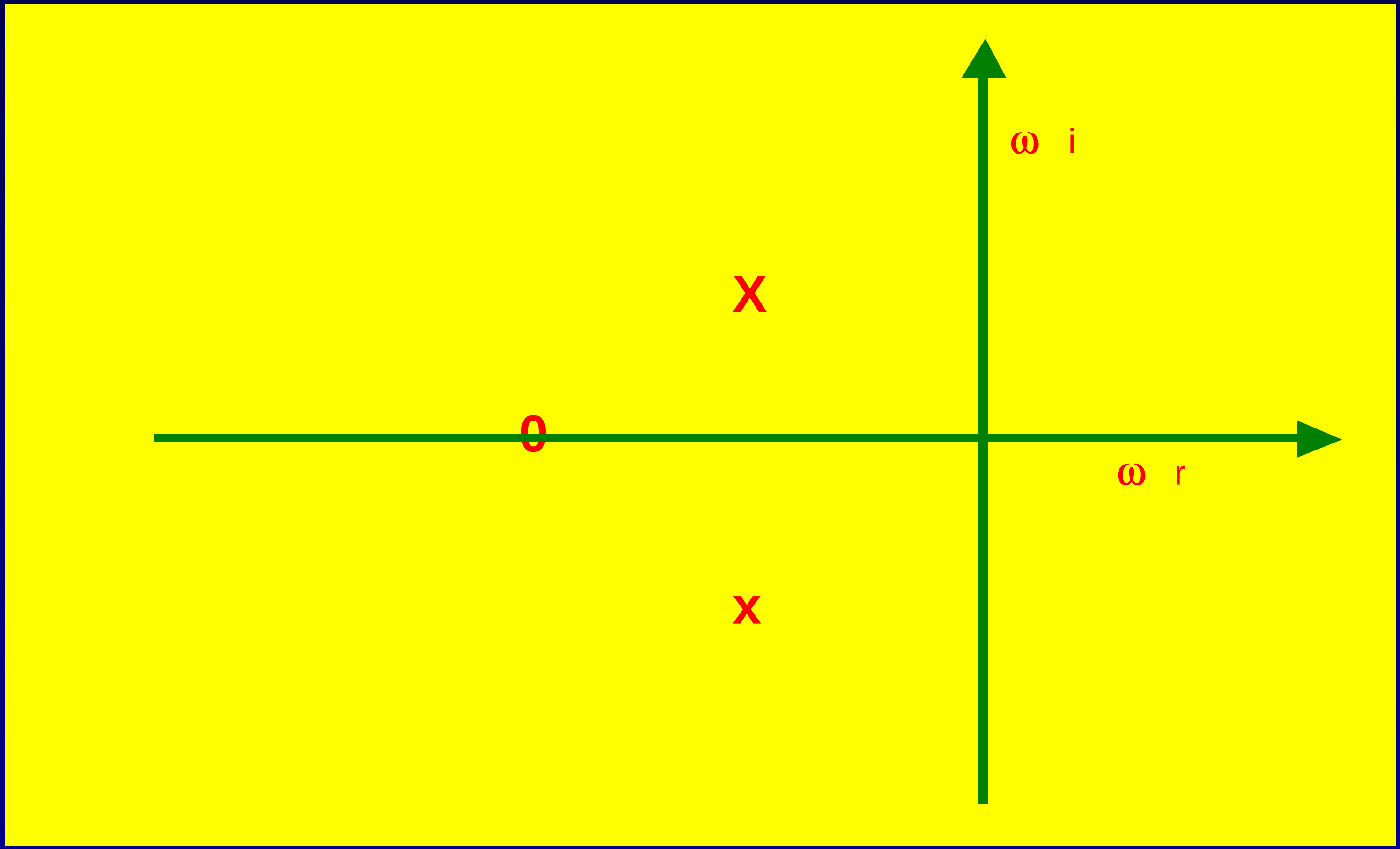
PLL Poles and Zeros



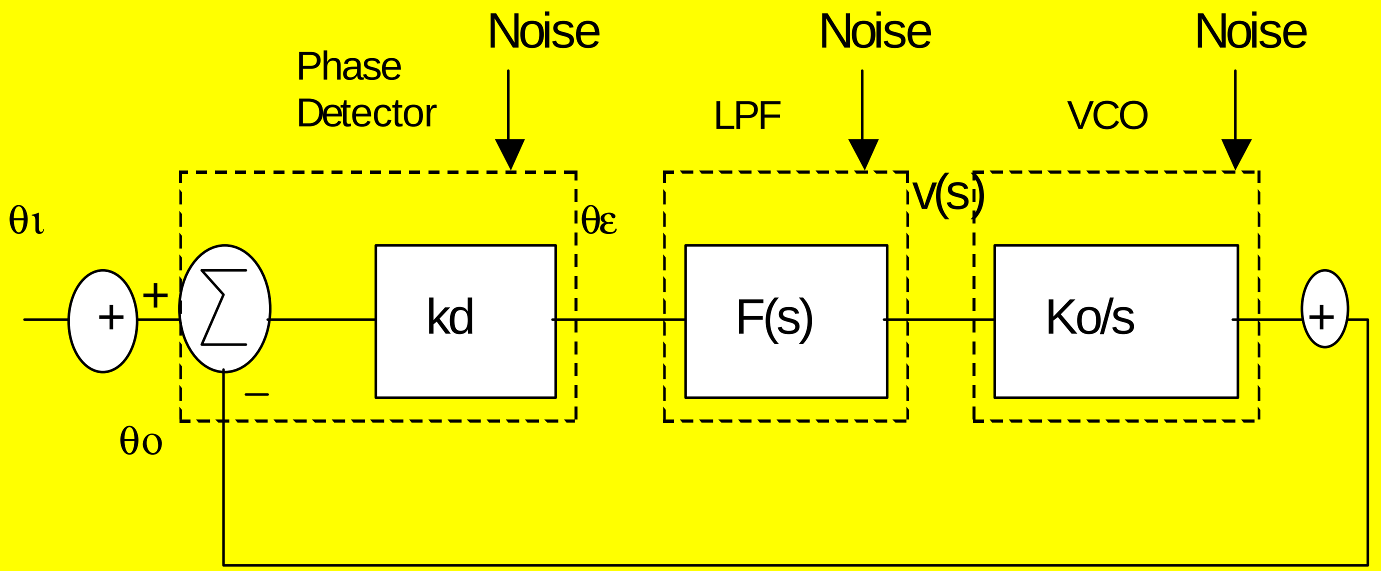
PLL Key Parameters

- Damping factor
- Natural frequency
- Locking time
- Locking range
- Pull-in time
- Pull-in range
- Noise bandwidth ….

PLL Noise Processes

- Thermal noise $S_{th,i} = 2kT/R$
- Short noise $S_{s,i} = qi(t)$
- flick noise $S_f = K_a \frac{I^a}{f_m{}^b}, b \sim 1$
- Random walk $S_f = K_a \frac{I^a}{f_m{}^b}, b \sim 2$
- High order random noise

PLL Noise Processes



PLL Noise Model

I.) Time-domain approach

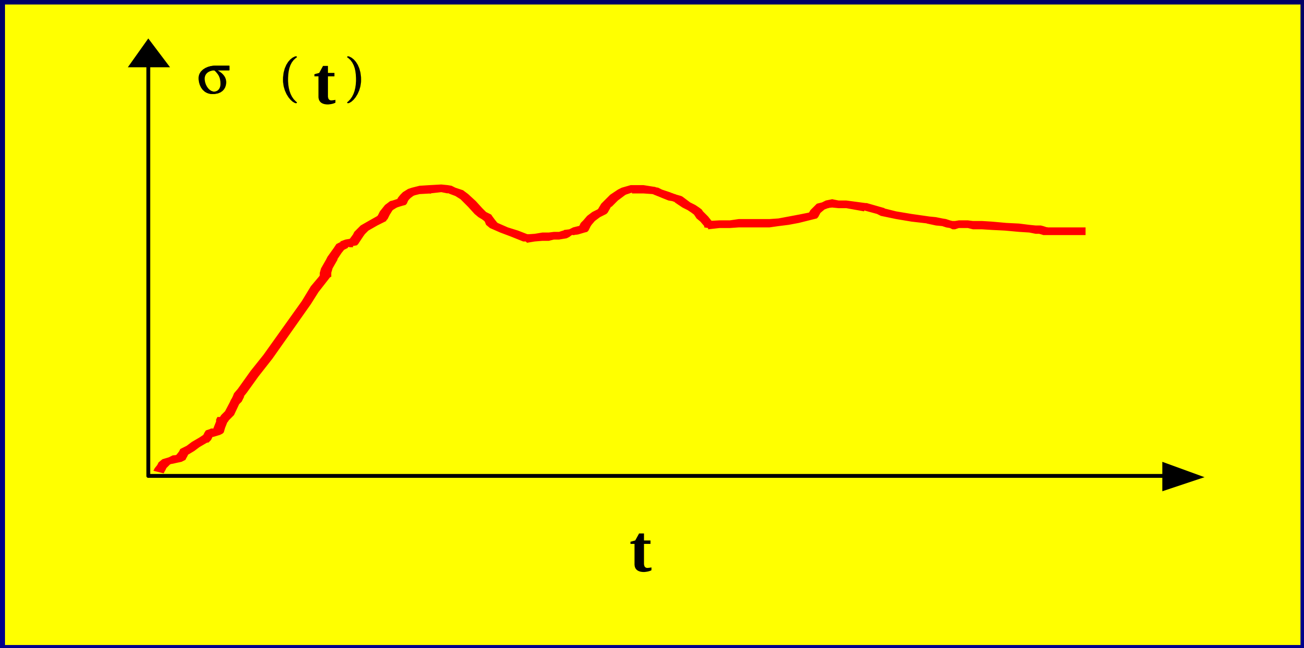
$$s_t^{2}(t) = 2(s_0^{2} - R_{tt}(\Delta t_n(t), \Delta t_0))$$

- But

$$R_{tt}(\Delta t_n(t), \Delta t_0) = \Im^{-1}(S(f))$$

PLL Noise

Variance (or sigma) record



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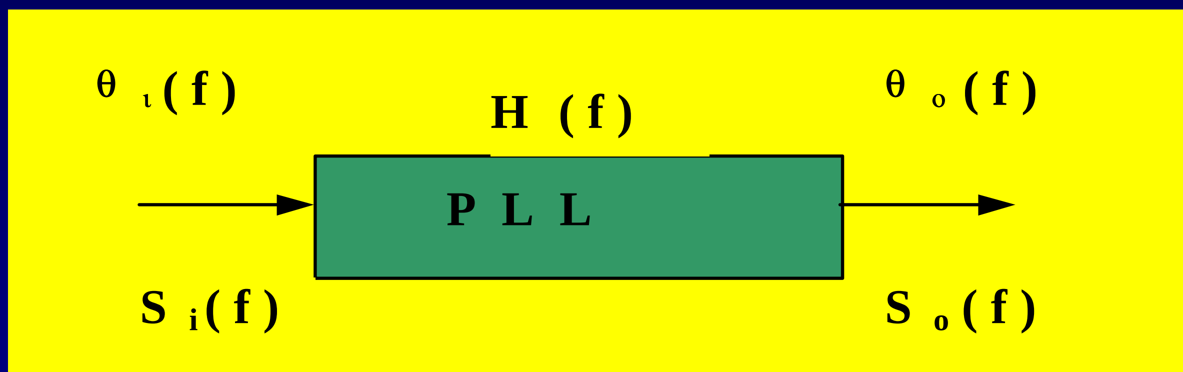
PLL Noise

Some basics on LTI system



$$q_o(f) = H_o(f)q_i(f)$$

$$S_o(f) = \left|H_o(f)\right|^2 S_i(f)$$

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PLL Noise Model

II.) Frequency-domain approach

$$S_o(f) = \sum_i S_i(f) \left| \frac{H_{FG\_i}(s)}{1 + H_{OL}(s)} \right|^2_{s = j2\pi f}$$

- Key insight: PLL noise PSD manifests both noise process and transfer functions

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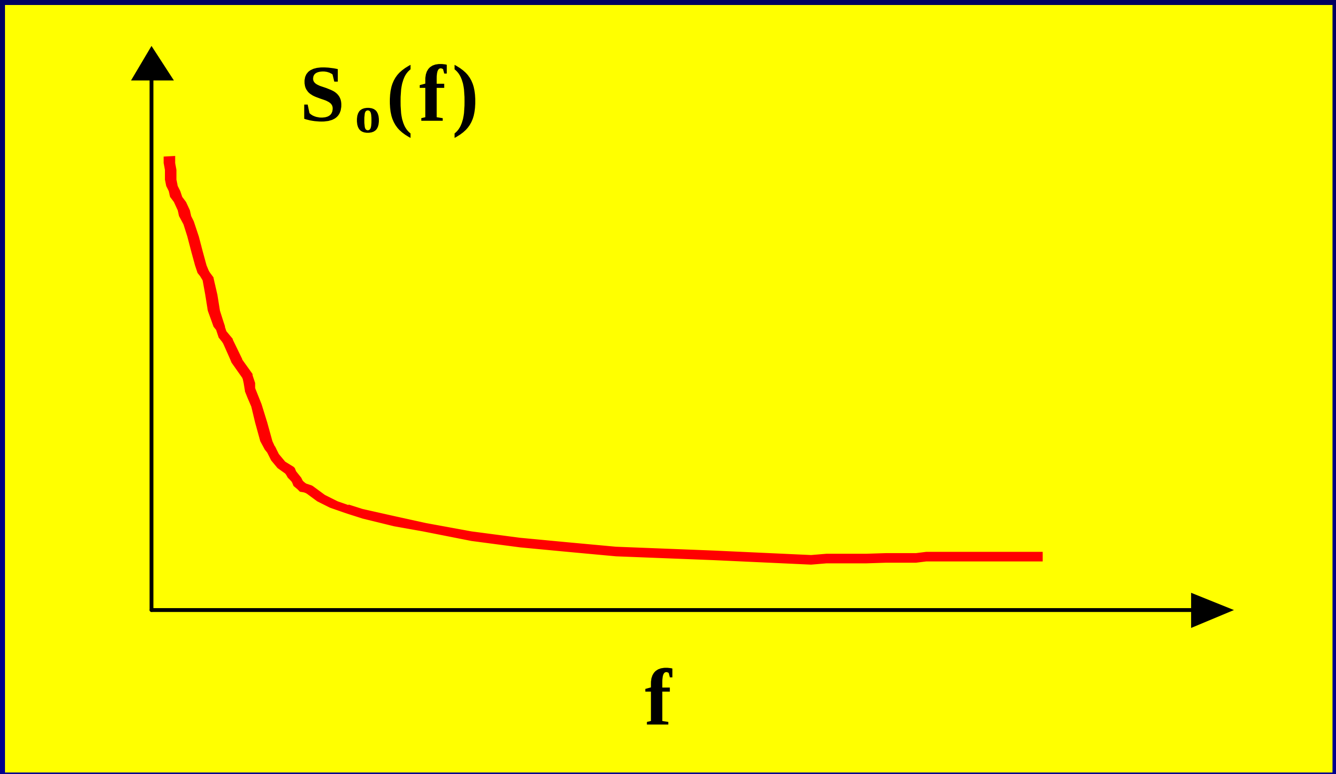
PLL Noise Spectrum

- Noise Power Spectrum Density



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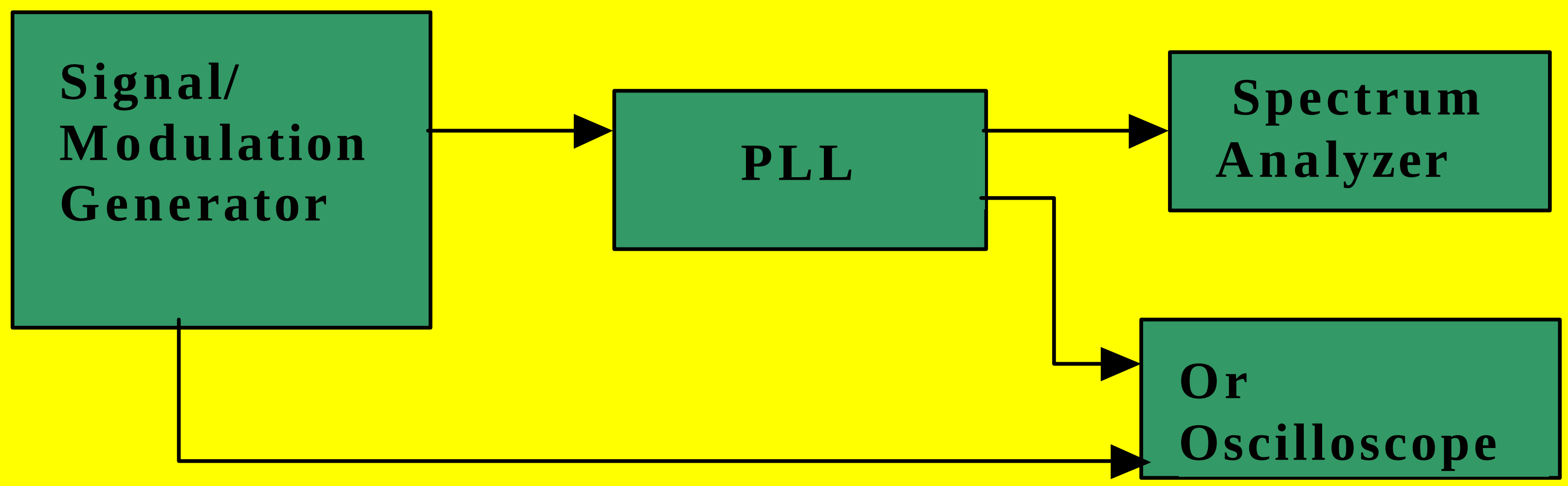
PLL Transfer Function and Gain Measurements

Traditional Methods



Limitations for Traditional Methods

- Requires a modulation & signal source
- Requires the access of PLL internal
- It is a piece-meal approach
- No separation of noise from transfer function
- It is slow
- No prediction capability

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New PLL Measurement and Analysis Method

- The methodology is based on the fact that PLL variance tracks both noise process and transfer function.
- The methodology takes the advantage of Time Interval Analyzer (TIA) that can take > 1 million measurements per second.
- The methodology determines the noise PSD and transfer function based on measured variance time record.

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Illustration of the New Method

- For a second-order PLL

$$H_0(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Variance function will be:

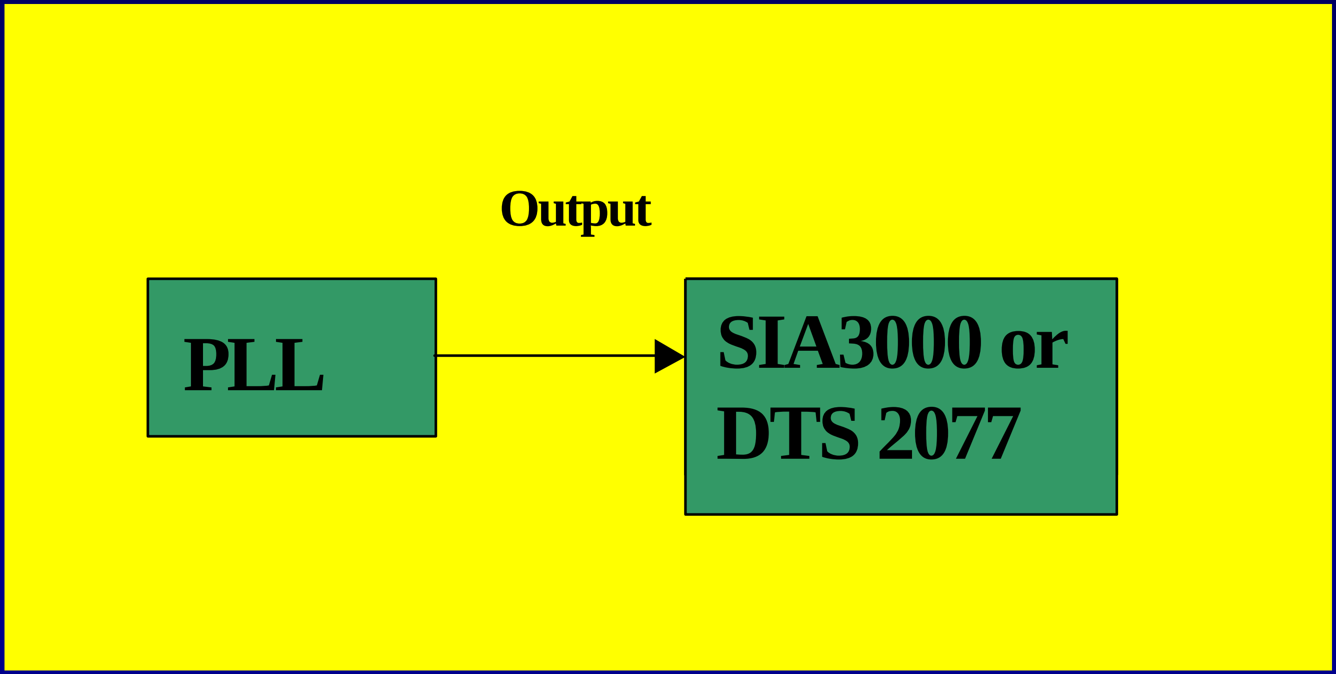
$$\sigma_t^2(\tau, \zeta, \omega_n, N_n)$$

- Parameters of ω_n, ζ, N_n are determined by

$$|\sigma_{t\_model}^2 - \sigma_{t\_measured}^2| < \varepsilon$$

A Case Study

- Setup



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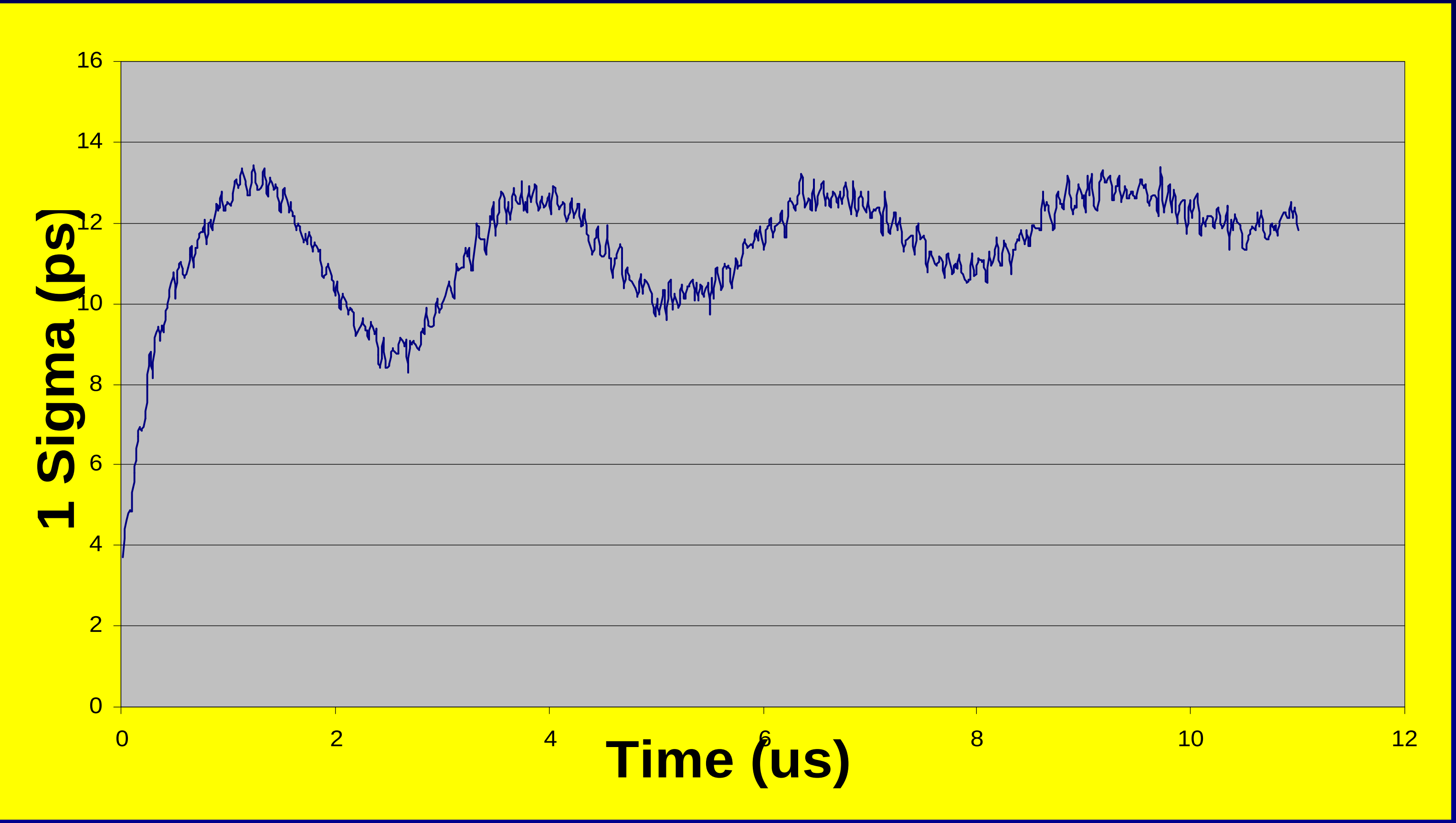
Variance Measurement Results



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PLL Transfer Function and Noise PSD

- Damping factor: $\zeta = 0.11$
- Natural frequency: $\omega_n = 4.30$ MHz
- Average PSD: $N_n = 3.16 \times 10^{-7}$ μw/Hz

$$H_0(s) = \frac{0.906\, s + 18.49}{s^2 + 0.906\, s + 18.49}$$

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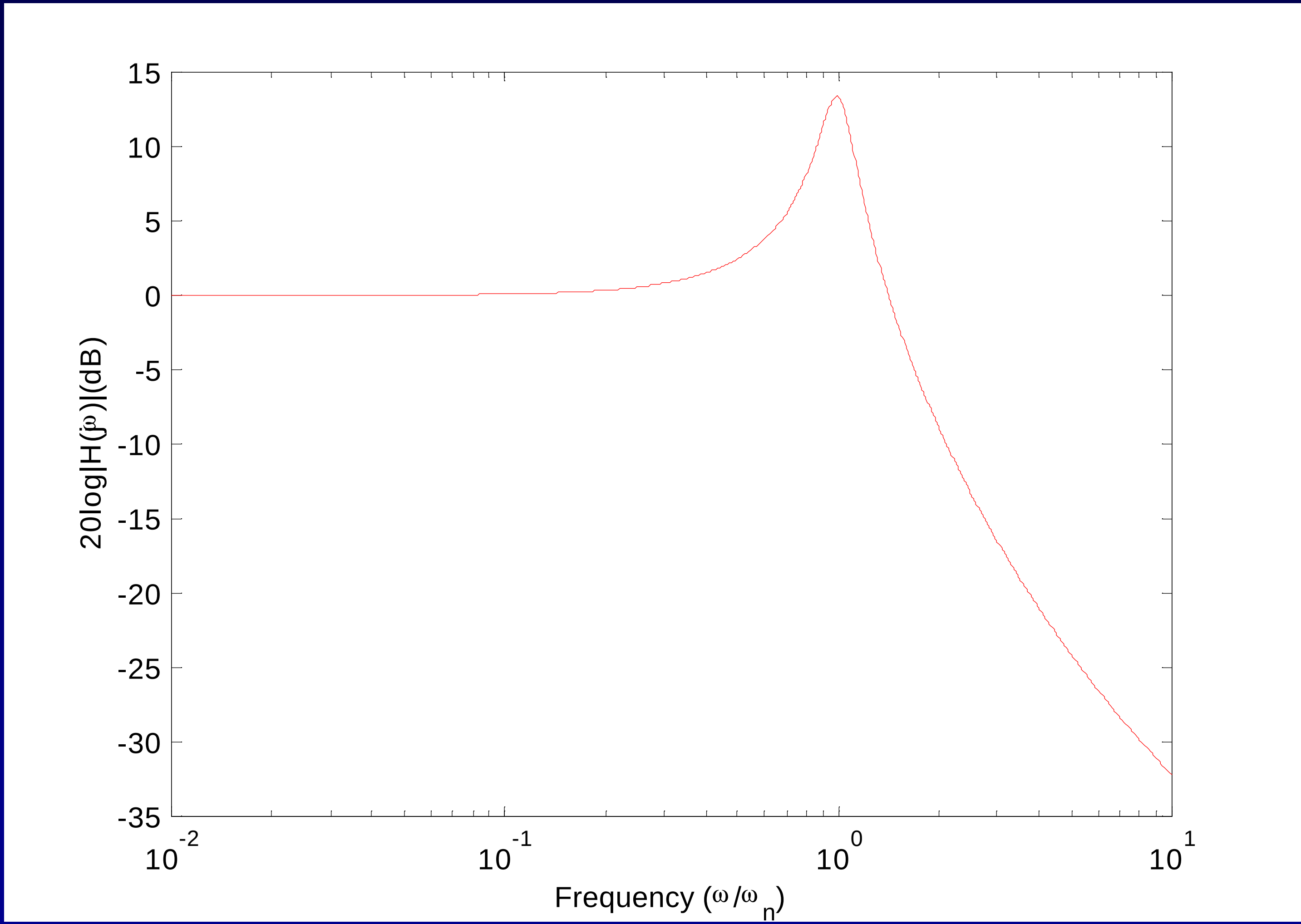
PLL Transfer Function



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PLL Parameters

- Lock-in time: $T_L = \frac{2p}{w_n}$
- Lock range: $\Delta w_L \approx 2zw_n$
- Pull-in time: $T_P = \frac{p^2}{16}\frac{\Delta w_0^2}{zw_n^3}$
- Noise bandwidth : $B_L = \frac{w_n}{2}\left(z + \frac{1}{4z} \right)$

etc.....

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Analysis Functionalities

- Pole/zero locations
- Bode plots
- Root locus
- Stability analysis
- "*in situ*" simulation and prediction

New Measurement Platform: **SIA3000**

- Up to 10 channels (single ended or differential)
- >1 million measurements per second
- 3.2 GHz, 3.2 Gb/s speed
- 200 fs resolution
- < 2 ps rms noise floor
- ………..

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Conclusion

- A new theory links PLL transfer function with noise processes.
- A methodology measures and analyzes PLL transfer function and noise PSD in one pass.
- A methodology that is fast (~ second throughput) and does not require a stimulator.
- A methodology provides all the PLL parameters and functions.
- A methodology that makes compliance testing practical for PLL transfer function.

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