New Phase Locked- Loop (PLL) Measurement and Analysis Methods (2nd – 3rd Order) Without A Stimulus

Mike Li, Wavecrest

Jun Ma, Wavecrest

Mark Marlett, LSI Logic

LSI LOGIC

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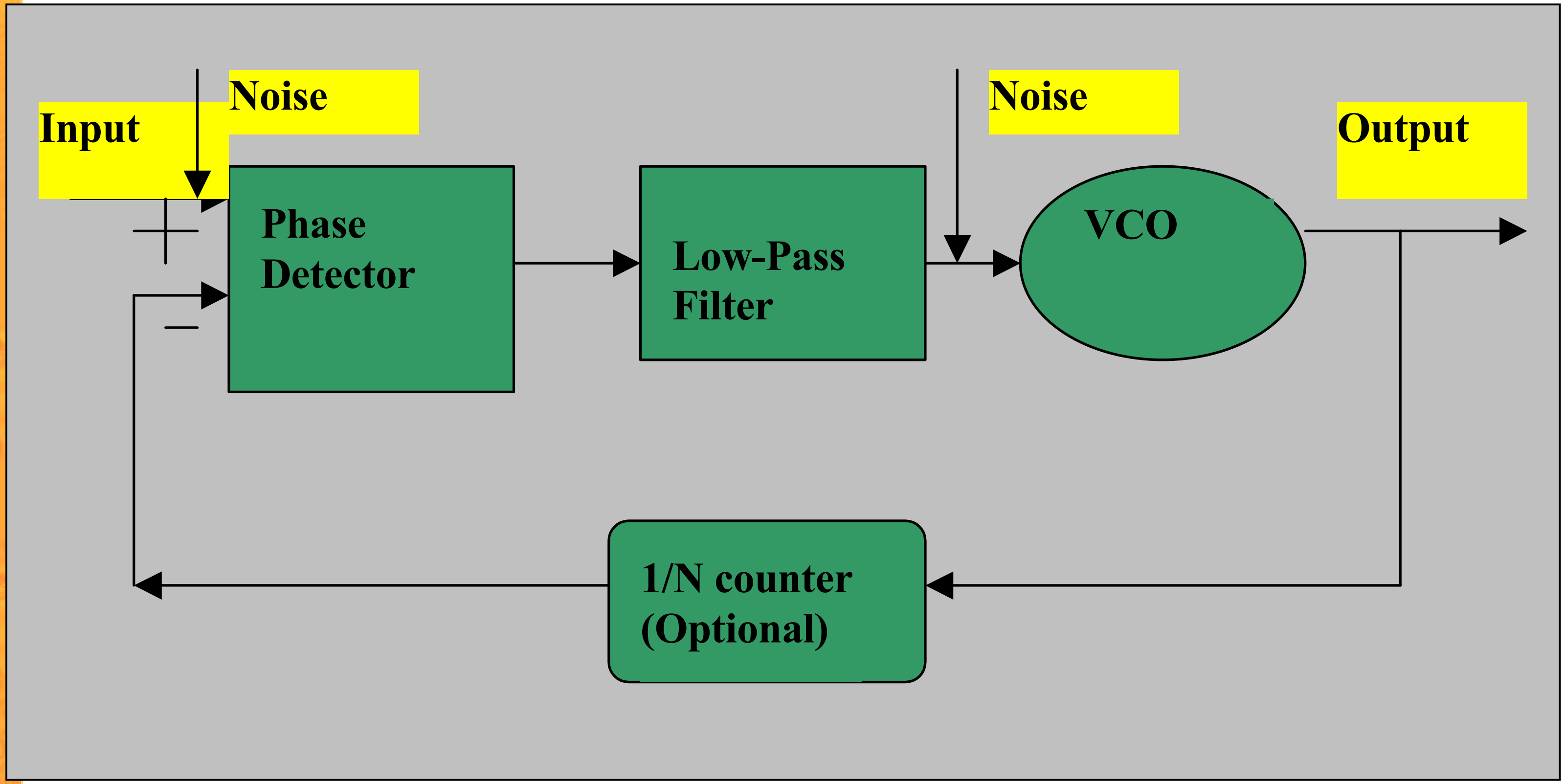
Purposes

- Review the autocorrelation/variance function based PLL analysis and 2nd order results
- Develop a generic system transfer function measurement and analysis method for a 3rd or Nth order PLL
- Simulation and experiment that verifies and proves the proposed method
- Apply the new 3rd or Nth order PLL method to practical devices used in applications such as PCI Express and FB DIMM

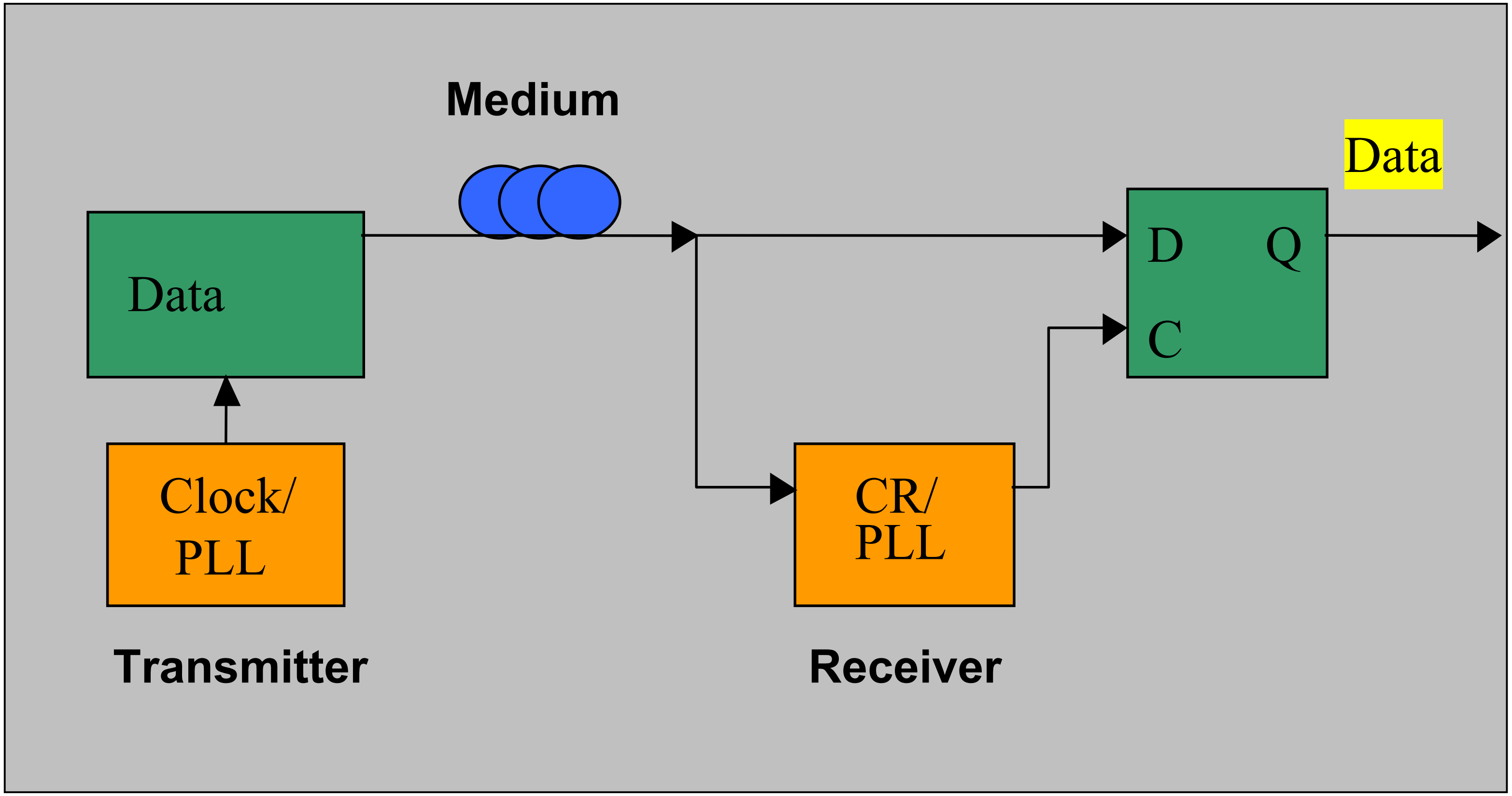
Outline

- **PLL applications**
- 2^{nd} PLL analysis review
- Third order PLL transfer function
- Performance analysis of the 3rd order PLL system
- Frequency domain based measurement approach
- Experiment and verification
- Conclusions and remarks

Phase-Locked Loop



PLL Application in Data Communications



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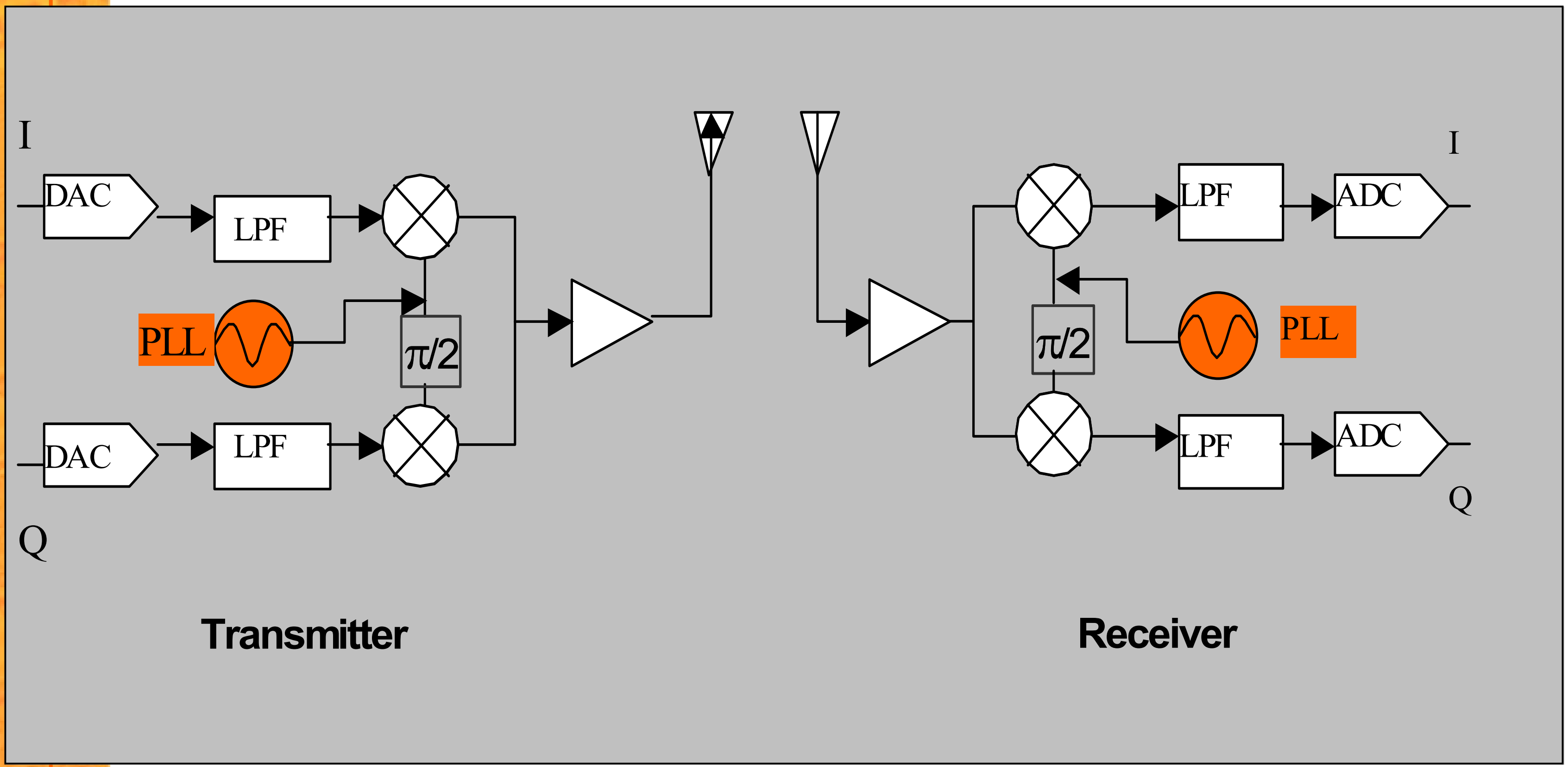
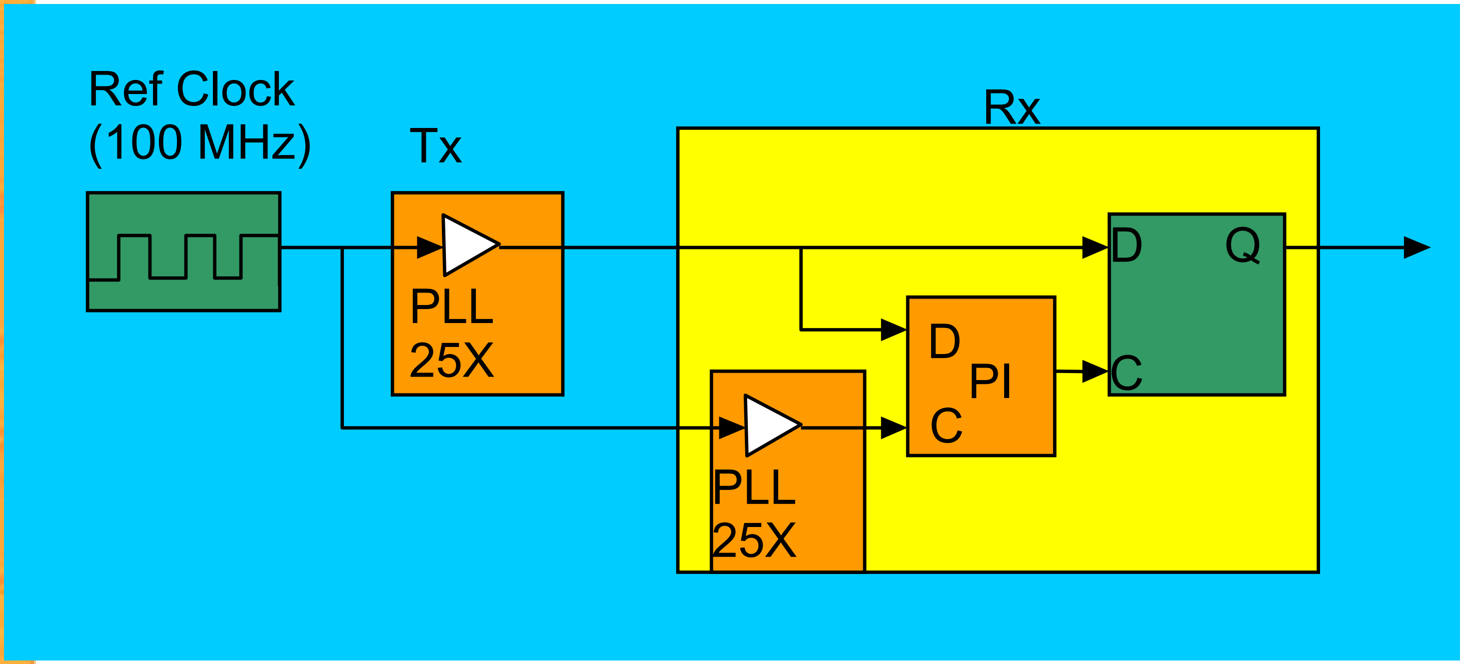
PLL Application in Wireless Communications



PCI Express and FB DIMM



- Testing PLL ***3 dB frequency*** and ***peaking*** is required by specification

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PLL Jitter Generic Model and Theory

Auto-correlation/Variance function and PSD

$$\sigma_t^2(t) = 2(\sigma_0^2 - R_{tt}(\Delta t_n(t), \Delta t_0))$$

but

$$R_{tt}(\Delta t_n(t), \Delta t_0) = \Im^{-1}(S(f))$$

and

$$\sigma_t^2(t) = 2(\sigma_0^2 - \Im^{-1}(S(f)))$$

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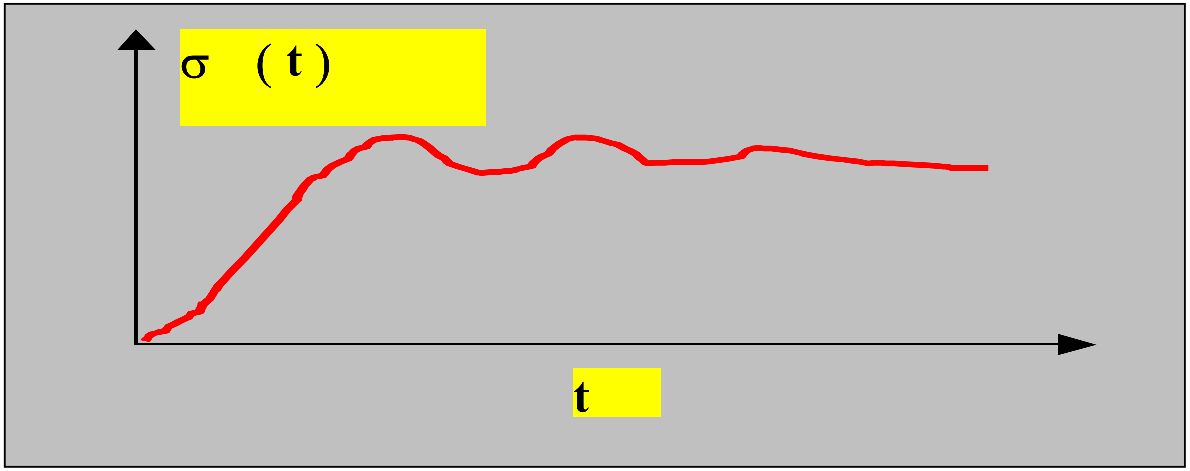
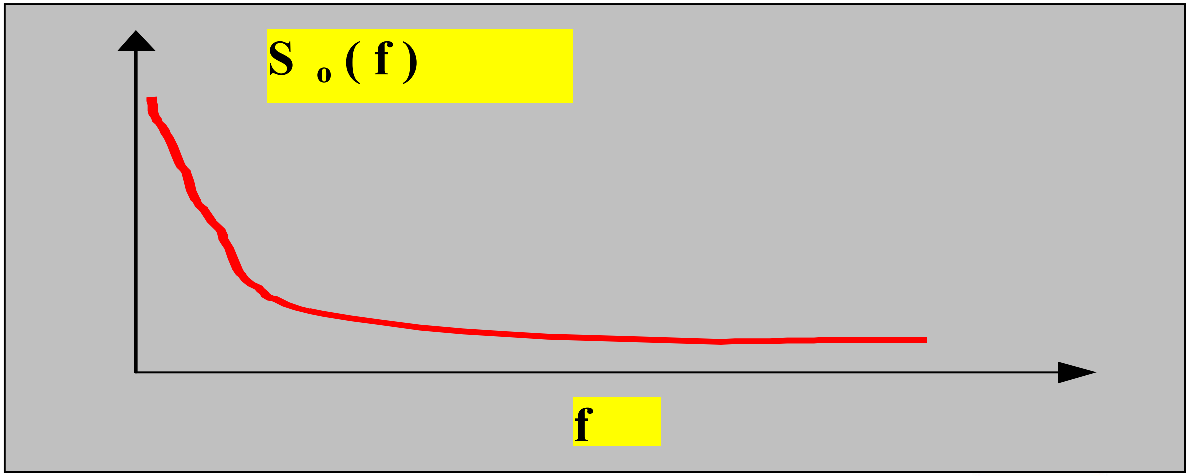
General Behavior of PLL Variance Function and Noise PSD

Variance



Noise PSD



2nd Order PLL Measurement Method

For a second-order PLL

$$H_0(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

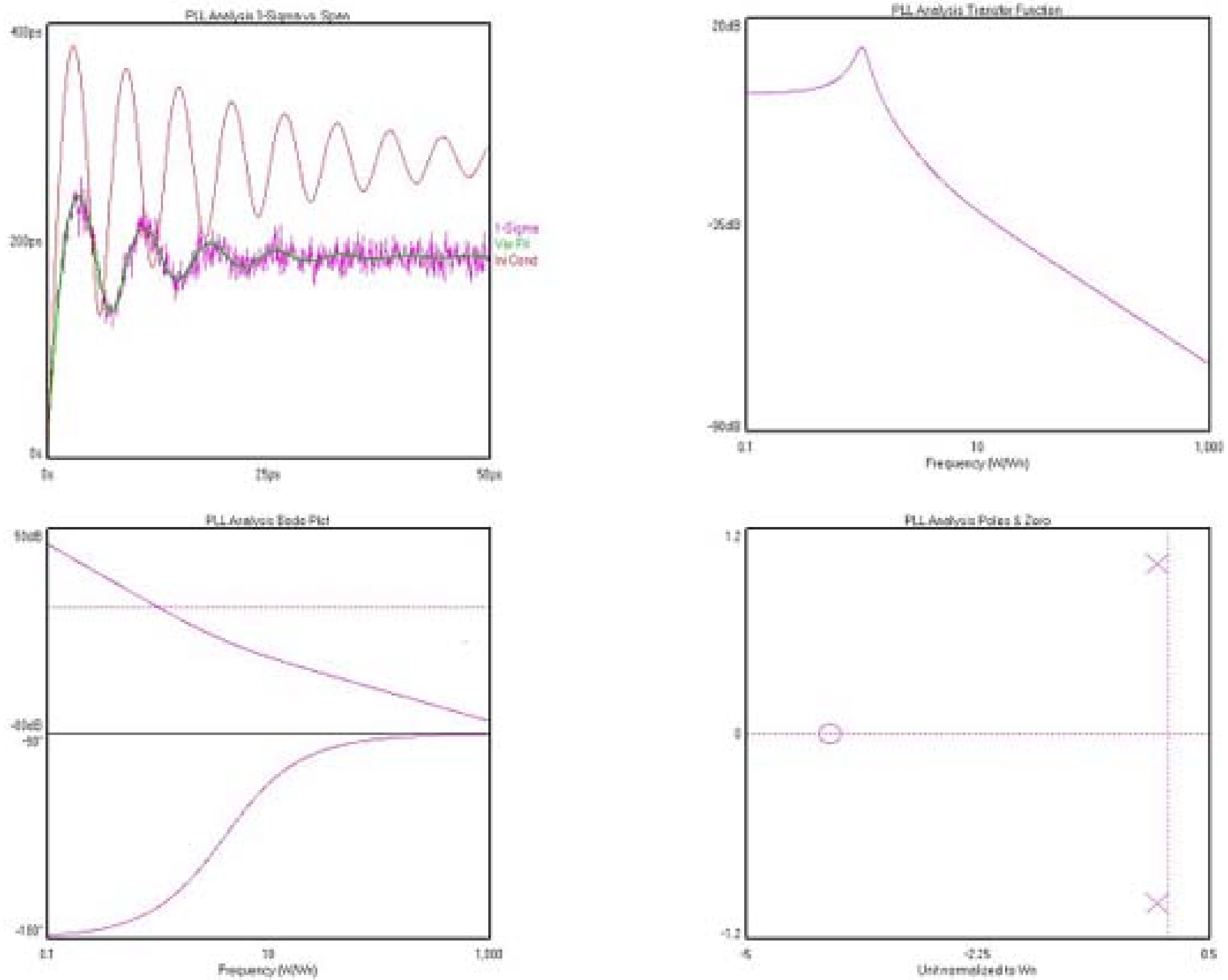
Variance function will be:

$$\sigma_t^2(t, \zeta, \omega_n)$$

Parameters of ω_n, ζ are determined by minimizing

$$|\sigma_{t\_model}^2 - \sigma_{t\_measured}^2| < \varepsilon$$

2nd Order PLL Measurement Results



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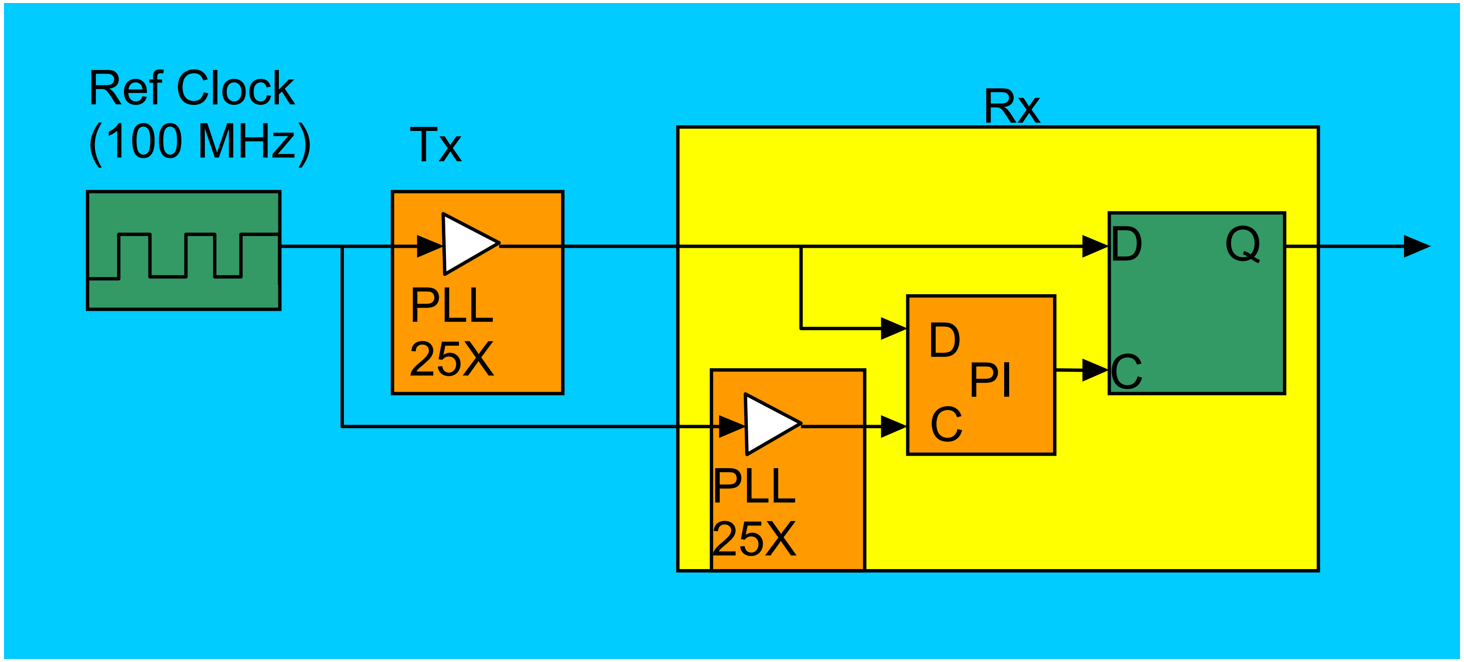
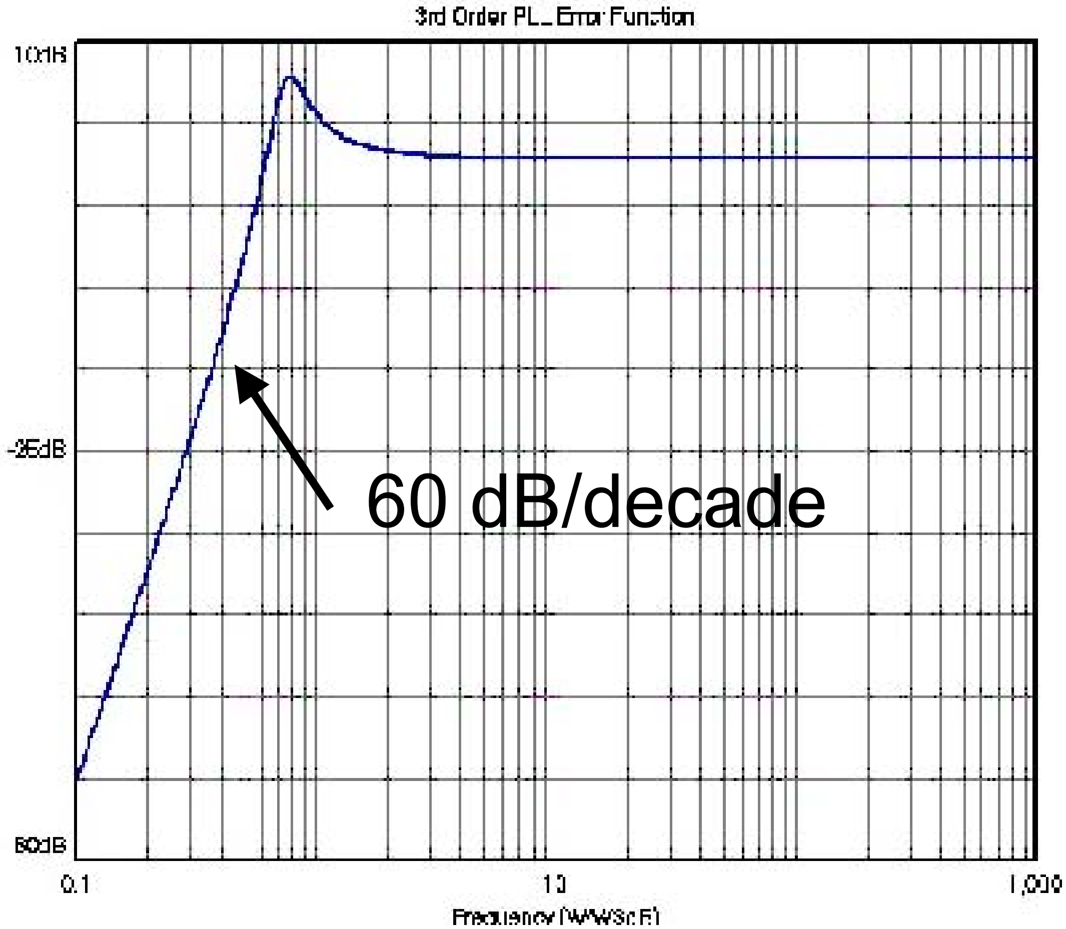
Why 3rd order PLL?



- Feed-through clock suppressing
- Phase error acceleration tracking
- Better jitter tracking at low frequency

A 3rd order PLL Jitter Transfer Function



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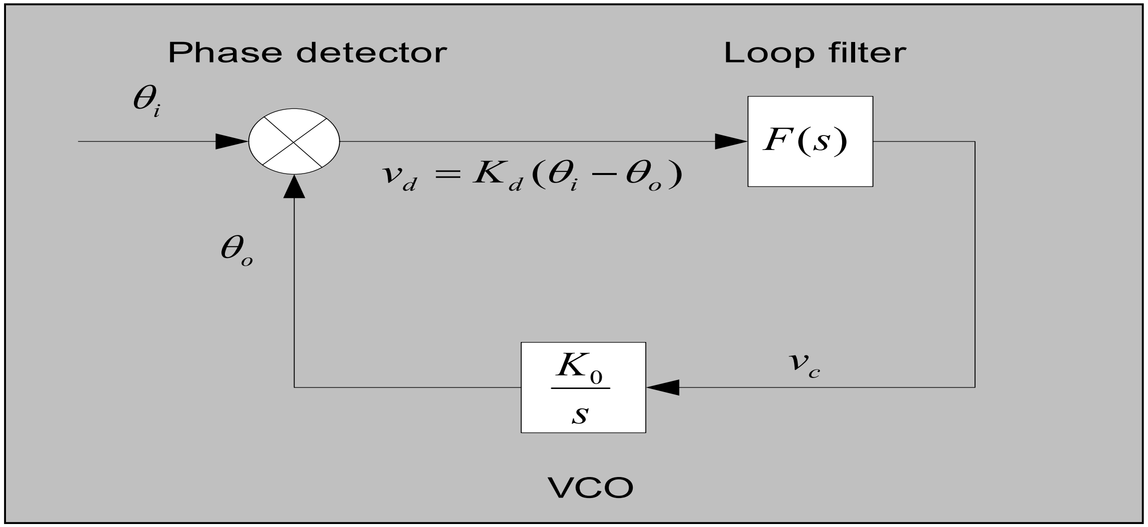
3rd Order PLL system



Open loop transfer function:

$$G(s) = \frac{K_0 K_d F(s)}{s}$$

Closed loop transfer function:

$$H(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{G(s)}{1 + G(s)} = \frac{K_0 K_d F(s)}{s + K_0 K_d F(s)}$$

Phase error:

$$\theta_e(s) = (1 - H(s))\theta_i(s) = \frac{s\theta_i(s)}{s + K_0 K_d F(s)}$$

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Steady State Response of the 3rd Order PLL

Final value theorem: $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$

Steady state phase error: $\lim_{t\to\infty} \theta_e(t) = \lim_{s\to 0} \frac{s^2\theta_i(s)}{s + K_0 K_d F(s)}$

A step change of *phase*: $\theta_i(t) = \Delta\theta\, u(t) \longleftrightarrow \theta_i(s) = \frac{\Delta\theta}{s}$

A step change of *frequency*: $\theta_i(t) = \Delta\omega t\, u(t) \longleftrightarrow \theta_i(s) = \frac{\Delta\omega}{s^2}$

A step change of *acceleration*: $\theta_i(t) = \frac{1}{2}\Delta\dot{\omega} t^2\, u(t) \longleftrightarrow \theta_i(s) = \frac{\Delta\dot{\omega}}{s^3}$

A Step Change of Acceleration (3^{rd} order PLL)

A step change of acceleration:

$$\theta_i(t) = \frac{1}{2}\Delta\dot{\omega}t^2\, u(t) \longleftrightarrow \theta_i(s) = \frac{\Delta\dot{\omega}}{s^3}$$

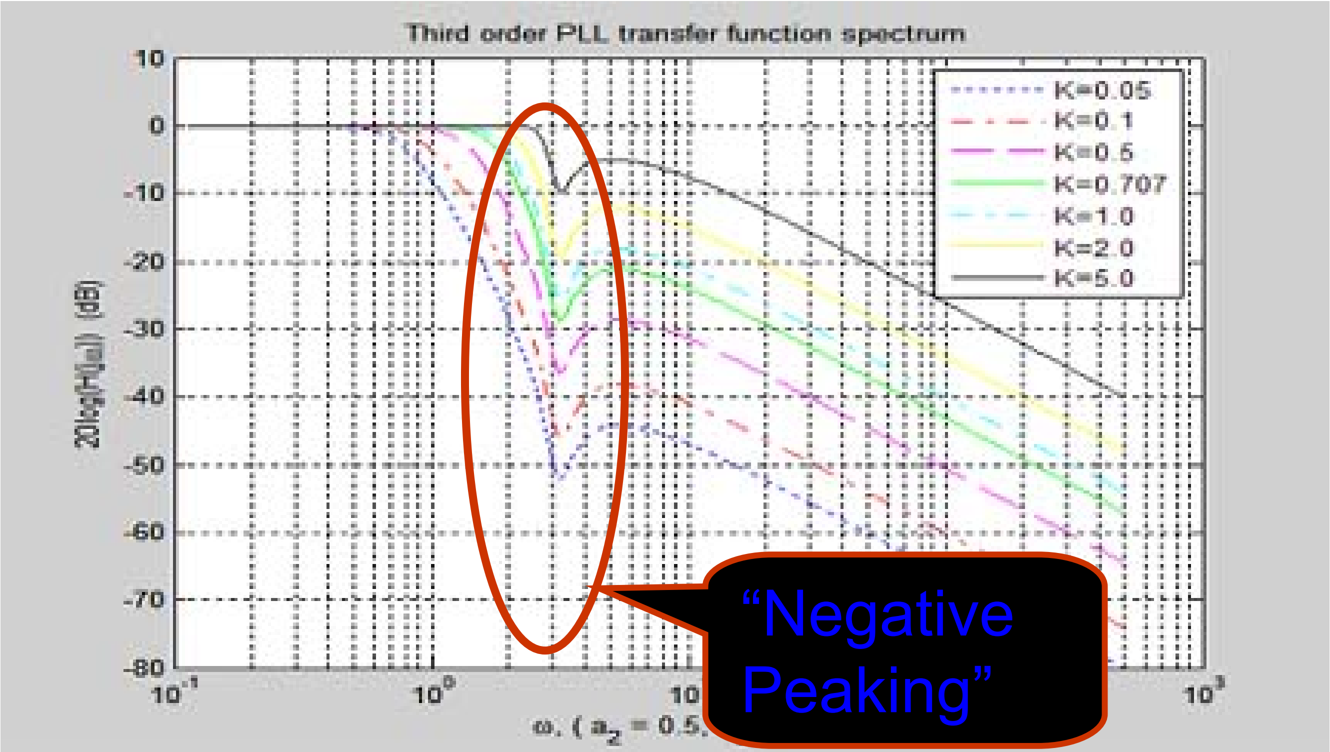
Steady state error:

$$\lim_{t\to\infty}\theta_e(t) = \lim_{s\to 0}\frac{s^2\theta_i(s)}{s + K_0 K_d F(s)}$$
$$= \lim_{s\to 0}\frac{\Delta\dot{\omega}}{s^2 + s\cdot K_0 K_d F(s)}$$

Track *acceleration step* change:

$$\lim_{t\to\infty}\theta_e(t) = 0 \quad\rightarrow\quad F(s) = \frac{s^2 + a_2 s + a_3}{s^2} \quad\rightarrow\quad 3^{rd} \text{ order PLL},\ H(s) = \frac{K(s^2 + a_2 s + a_3)}{s^3 + K(s^2 + a_2 s + a_3)}$$

3rd Order PLL Spectrum



$$H(s) = \frac{K(s^2 + a_2 s + a_3)}{s^3 + K(s^2 + a_2 s + a_3)}$$

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Noise Performance of 3rd Order PLL

PLL loop bandwidth:

$$B_L = \int_0^{\infty} \left| H(j2\pi f) \right|^2 df \qquad Hz$$

3rd order loop BW:

$$B_L = \frac{1}{4} K \cdot \frac{a_2 K + a_2^2 - a_3}{a_2 K - a_3} \qquad Hz$$

Phase jitter variance:

$$\sigma_{no}^2 = \frac{W_i B_L}{P_s} = \frac{2 N_o B_L}{V_s^2} \qquad rad^2$$

Loop SNR:

$$SNR_L = \frac{P_s}{2 B_L W_i} = \frac{V_s^2 / 2}{2 B_L N_o}$$

Tracking Performance of 3rd Order PLL

Linear tracking

– 3rd order PLL can track the step change of *phase, frequency,* and *frequency acceleration*.

Non-linear tracking

– *Hold range*: (*steady-state*)

$$\Delta\omega_H = \pm K_v = \pm K_0 K_d F(0)$$

– *Pull out range*: not much understanding (*transient*)

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Acquisition Performance of 3rd Order PLL

Phase acquisition

Lock in range : (loop gain)

$$\Delta\omega_L \approx \pm K = \pm K_0 K_d F(\infty)$$

Frequency acquisition

– *Pull in limit*:

$$\Delta\omega_P \cong \sqrt{2K_v K} \qquad K_v >> K$$

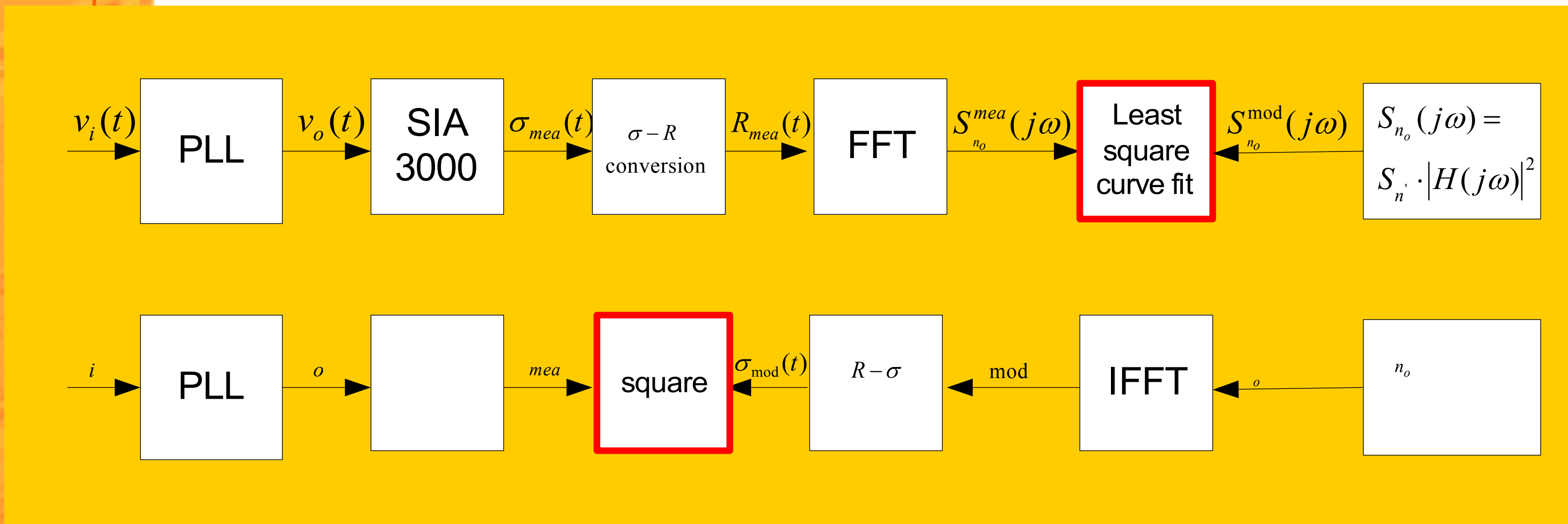
– *Pull in time*:

$$T_P \cong \sqrt{\frac{\pi}{a_3} \cdot \frac{\Delta\omega}{K}} \qquad \Delta\omega >> K$$

$$\Delta\omega_L < \Delta\omega_P < \Delta\omega_H$$

Lock in range < Pull in limit < Hold in range

Freq. vs. Time-Domain Optimization



$$H(s) = \frac{K(s^2 + a_2 s + a_3)}{s^3 + K(s^2 + a_2 s + a_3)}$$

$$R_{n_o}(t) = \sigma_0^2 - \frac{1}{2}\sigma^2(t)$$

$$\min_{\{K, a_2, a_3, S_{n'}\}} \varepsilon = \int_{-\infty}^{+\infty} \left| S_{n_o}^{mea}(j\omega) - S_{n_o}^{mod}(j\omega) \right|^2 d\omega$$

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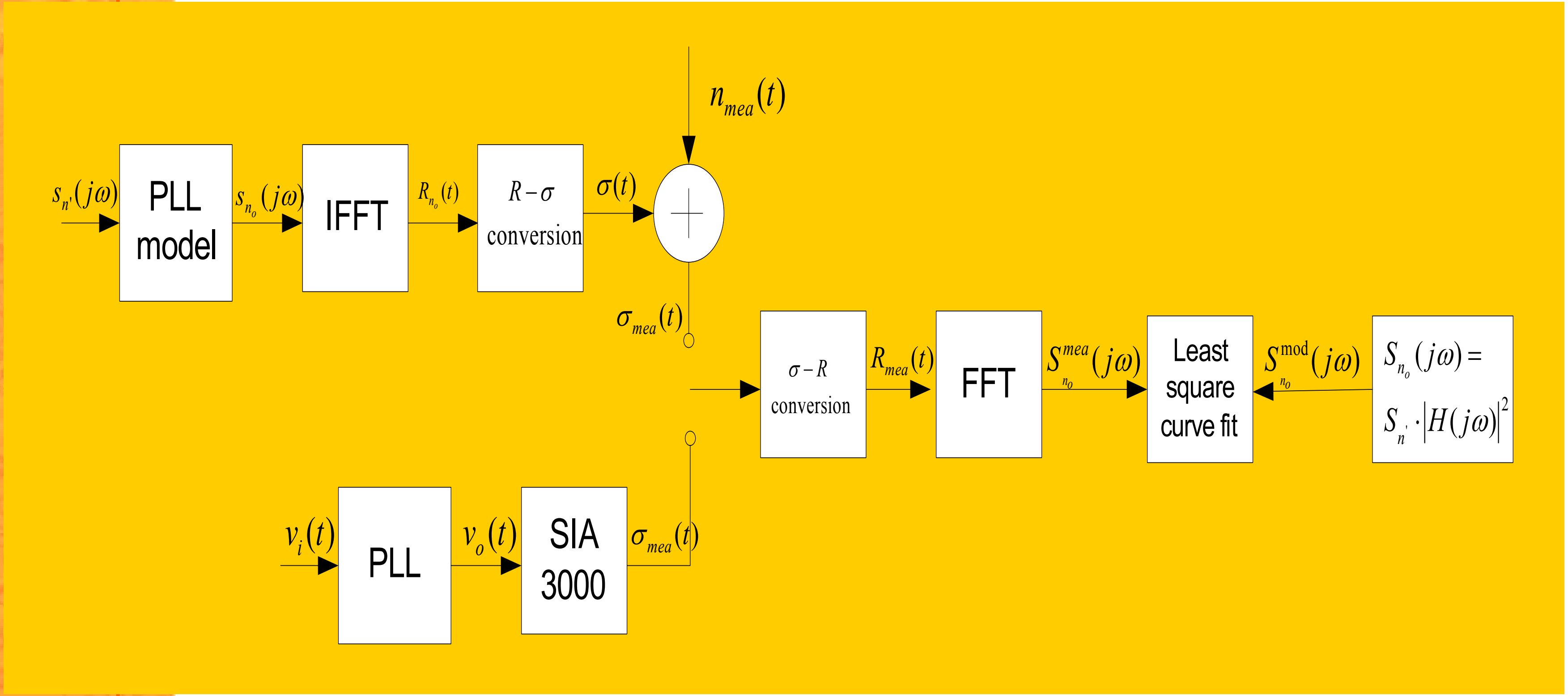
Simulation and Verification Model



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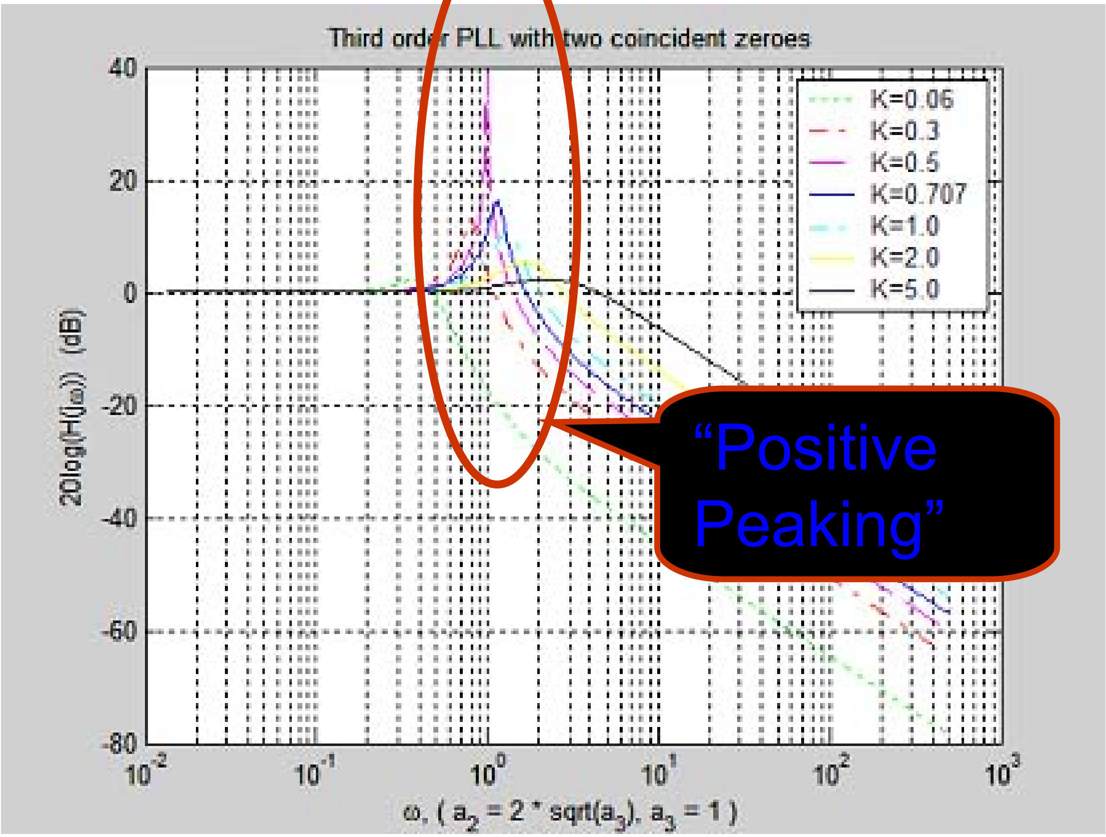
3rd Order PLL Transfer Function



$$H(s) = \frac{K(s^2 + 2s + 1)}{s^3 + K(s^2 + 2s + 1)}, \quad K = 0.707, K = 2$$

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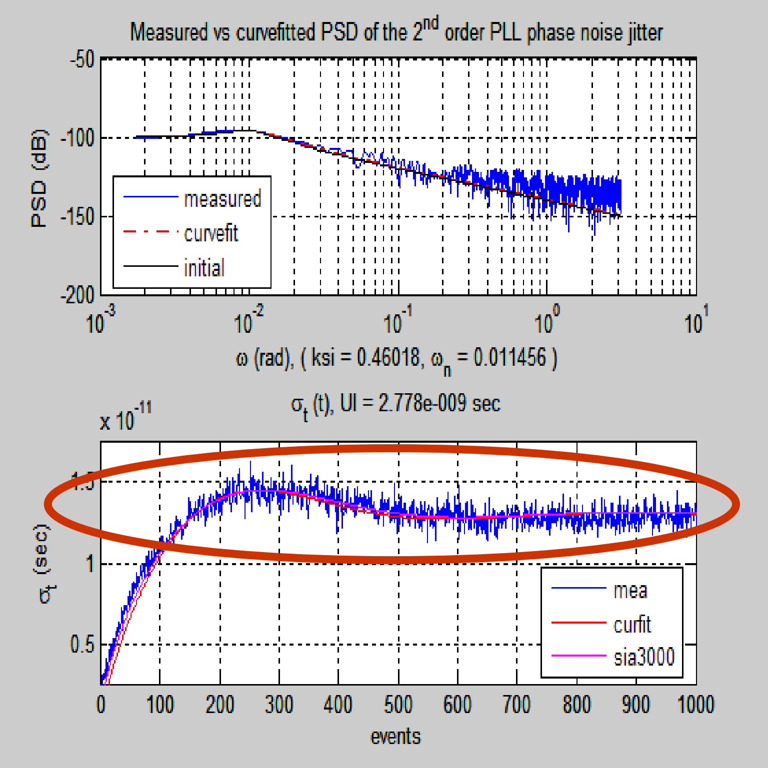
Lab Experiments and Verification Results

$$H(s) = \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

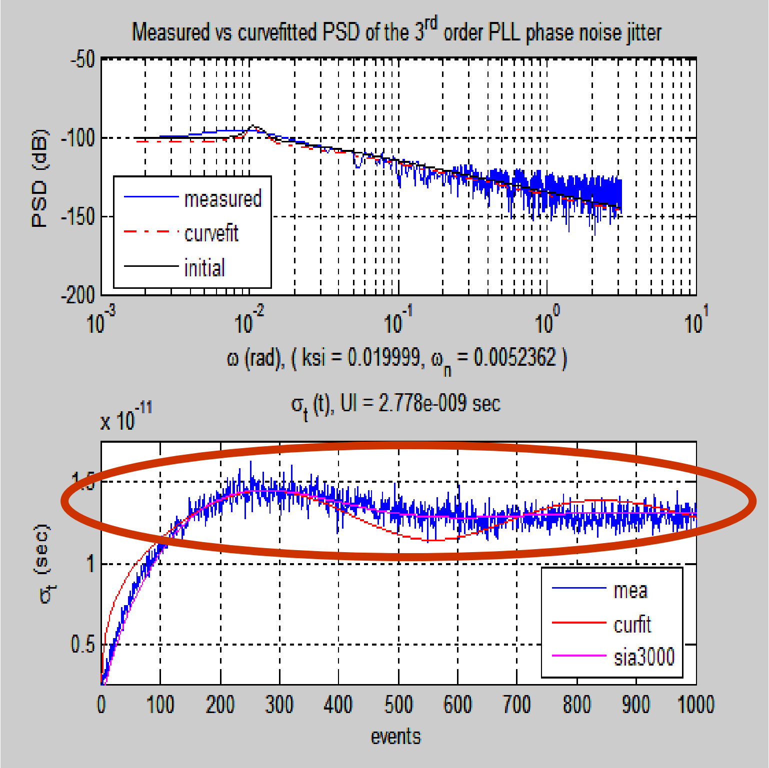
$$H(s) = \frac{K(s^2 + a_2 s + a_3)}{s^3 + K(s^2 + a_2 s + a_3)}$$

Cases	2nd Order		3rd Order		
F_c=4e6	ζ	ωn(rad)	K	a_2(rad)	a_3(rad)
A	**0.46**	**0.0108**	**0.02**	**0.0052**	**0.0353**
B	**0.36**	**0.0143**	**0.01**	**0.0175**	**0.0524**
C	**0.84**	**0.0654**	**0.07**	**0.0079**	**0.7886**
D	**1.00**	**0.0738**	**0.10**	**0.0094**	**0.1807**

Case A: 2nd and 3rd Order Results for a 2nd Order PLL device



2nd Order Model Fit



3rd Order Model Fit

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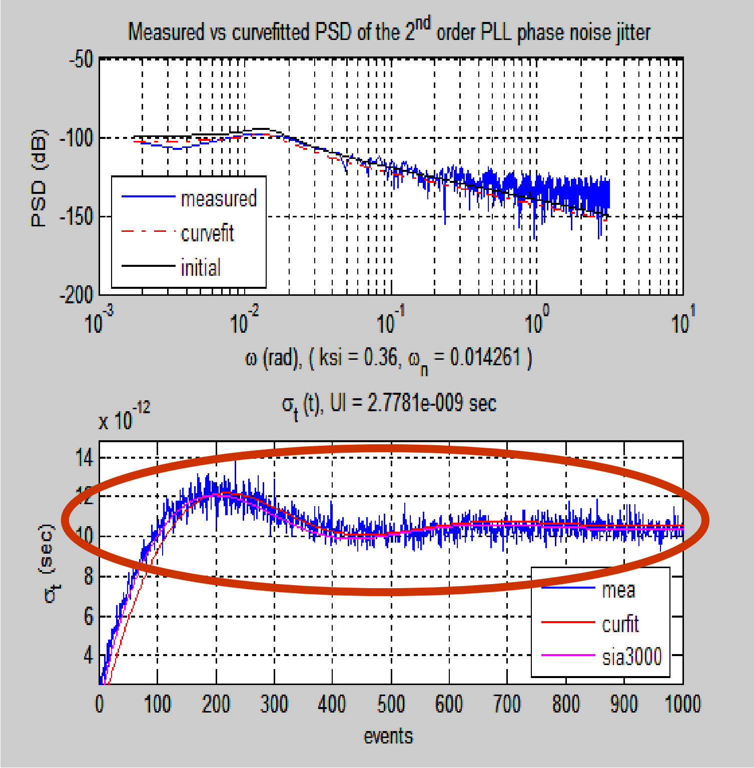
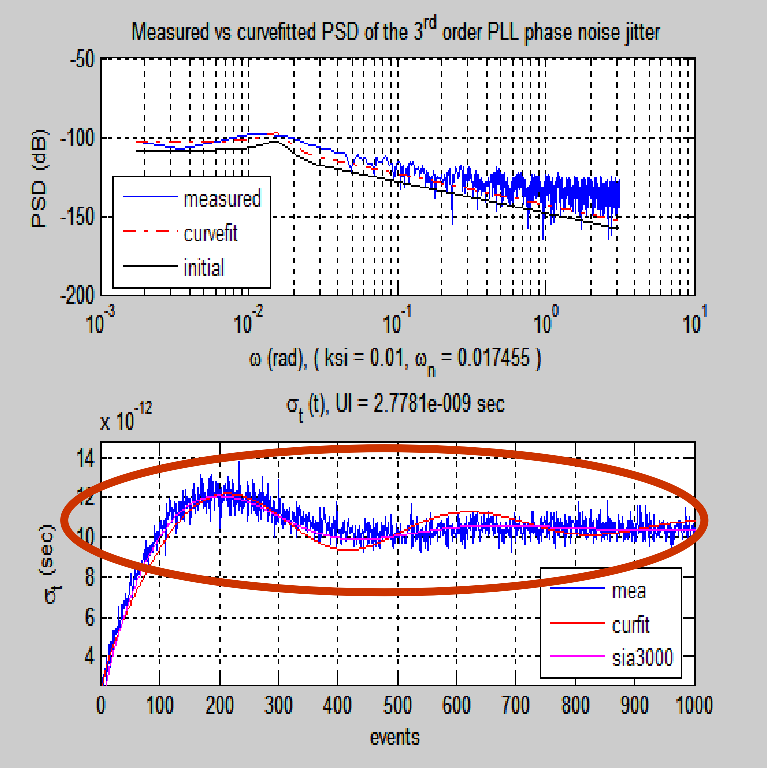
Case B: 2nd and 3rd Order Results for a Second 2nd Order PLL Device



2nd Order Model Fit



3rd Order Model Fit

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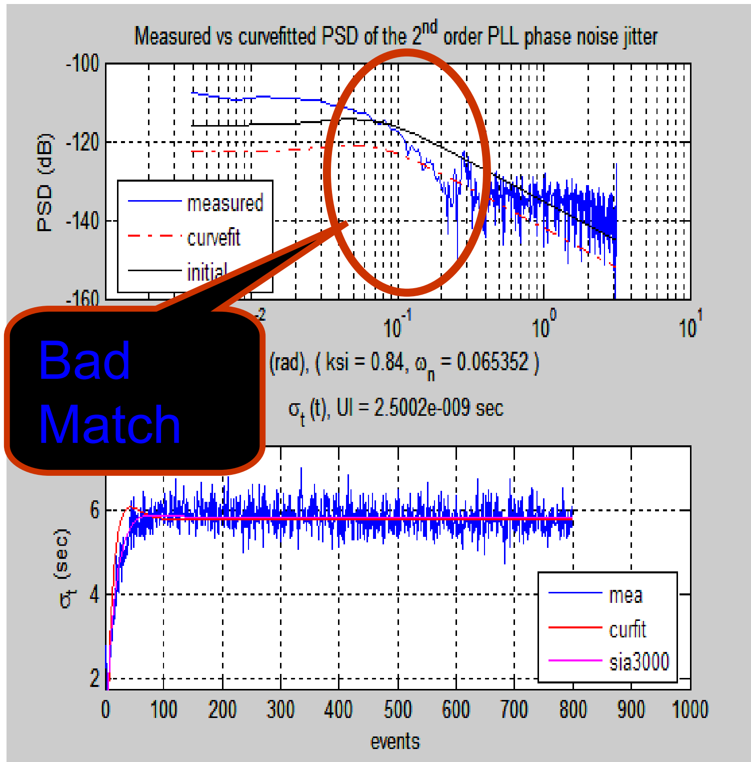
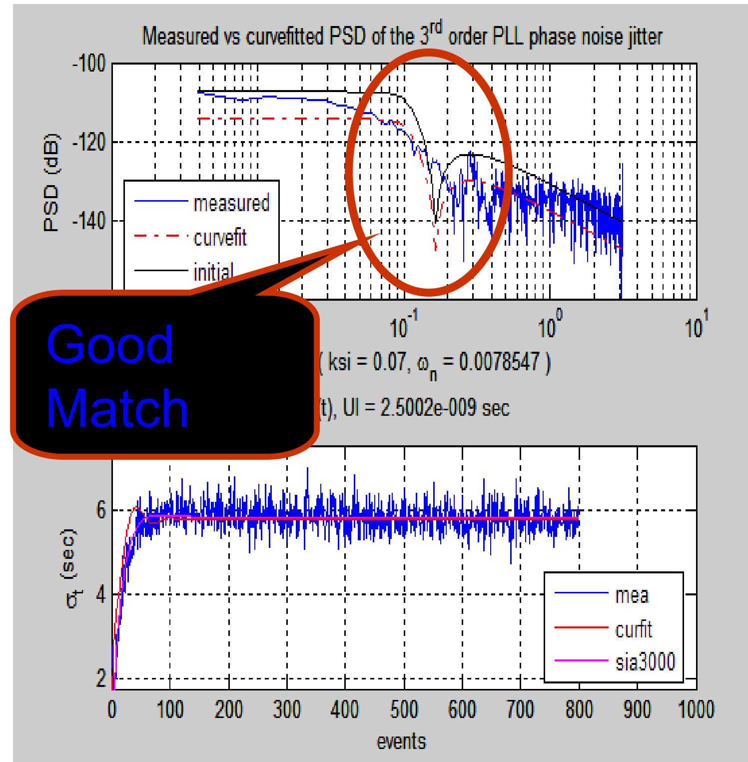
Case C: 2nd and 3rd Order Results for a 3rd Order PLL Device

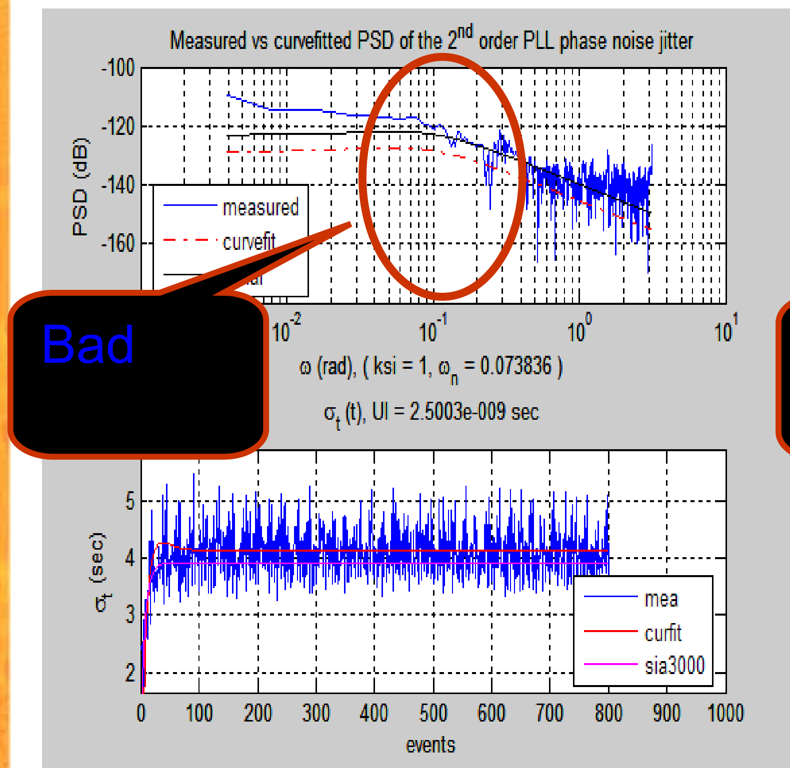


2nd Order Model Fit

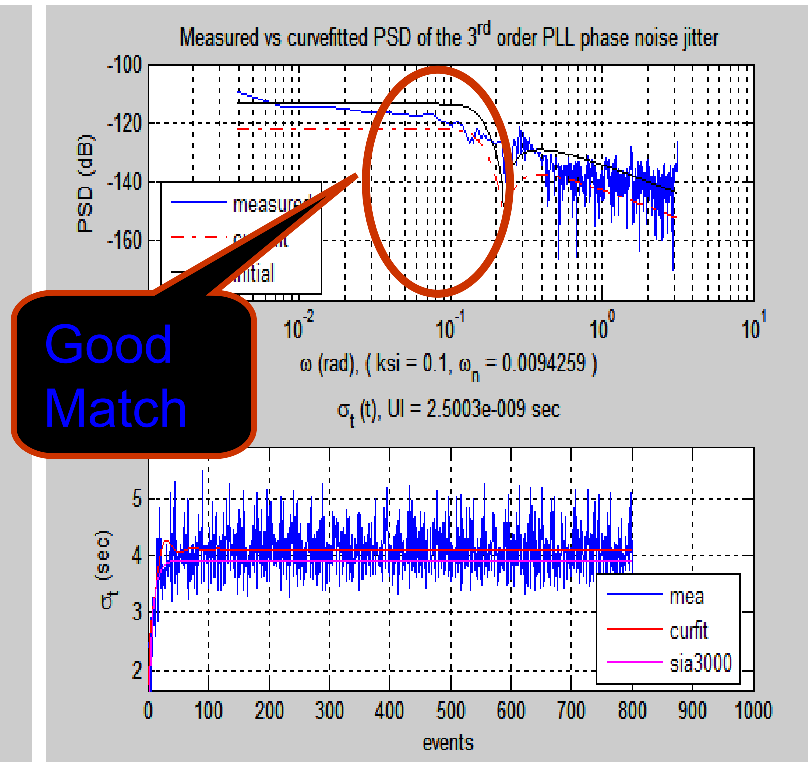


3rd Order Model Fit

Case D: 2nd and 3rd Order Results for a Second 3rd Order PLL Device

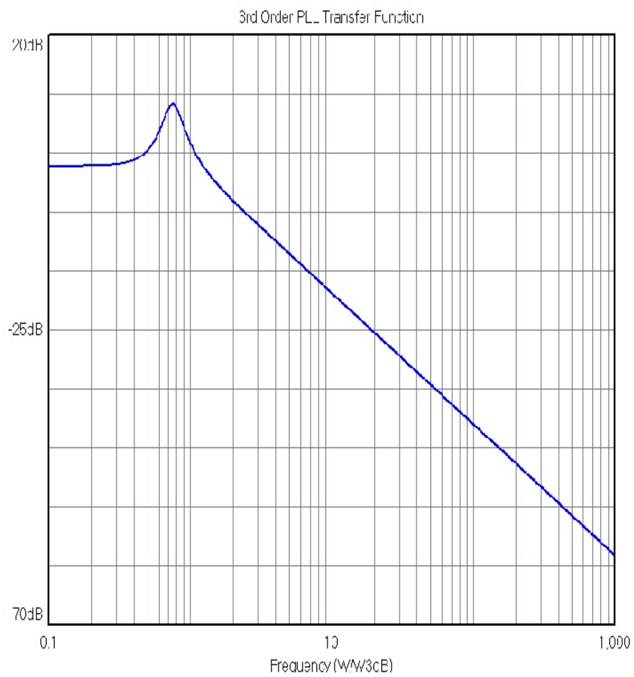


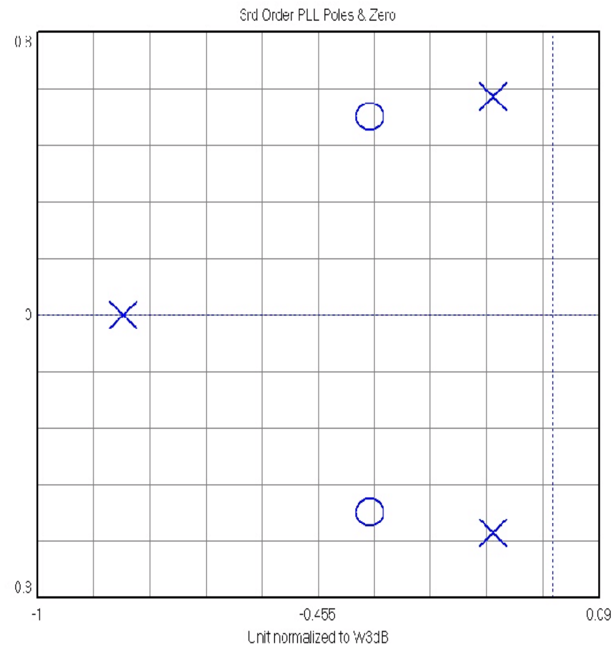
2nd Order Model Fit



3rd Order Model Fit

Additional Measurement Results for a 3rd Order PLL





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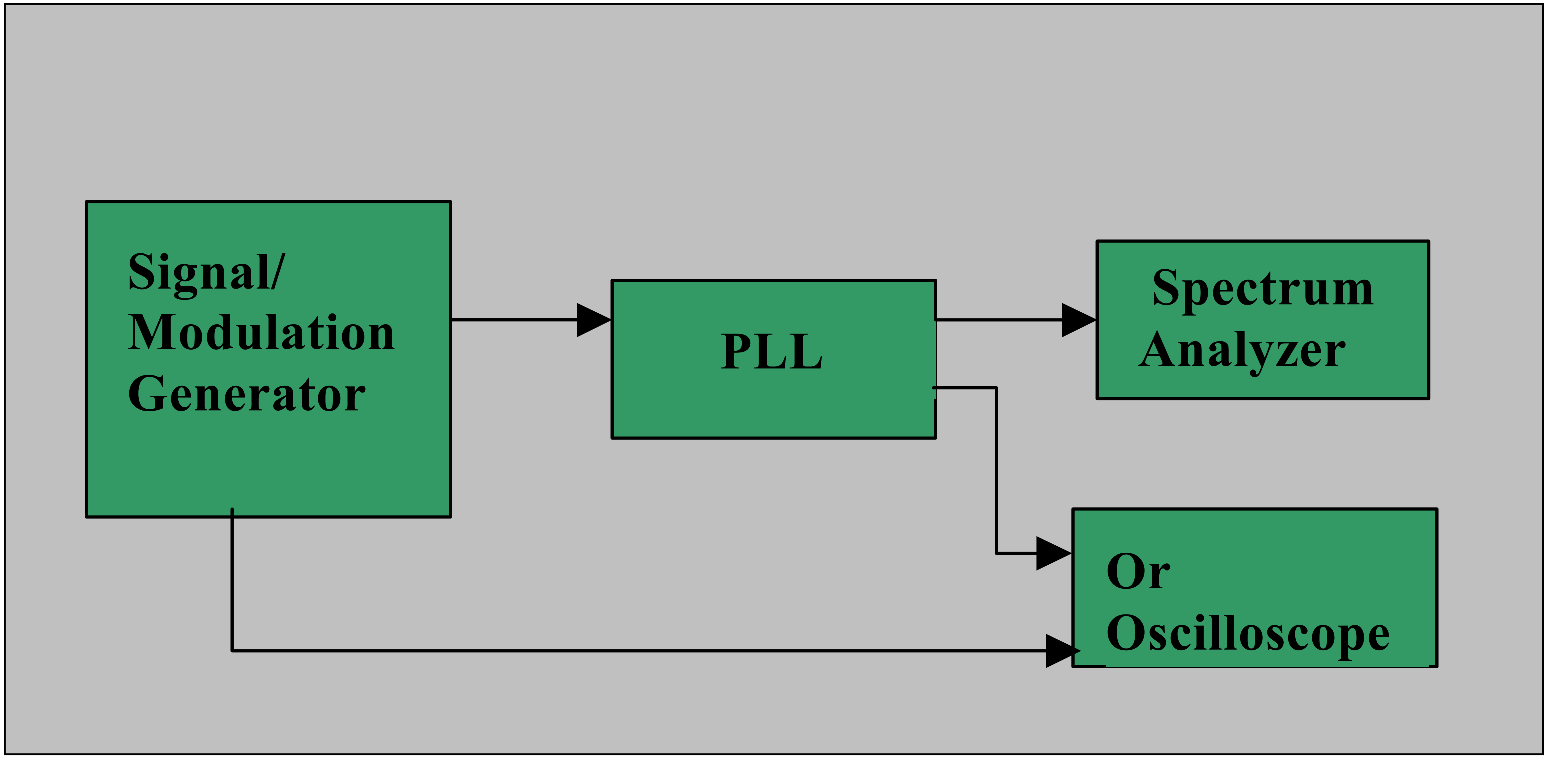
Traditional PLL Transfer Function Measurement



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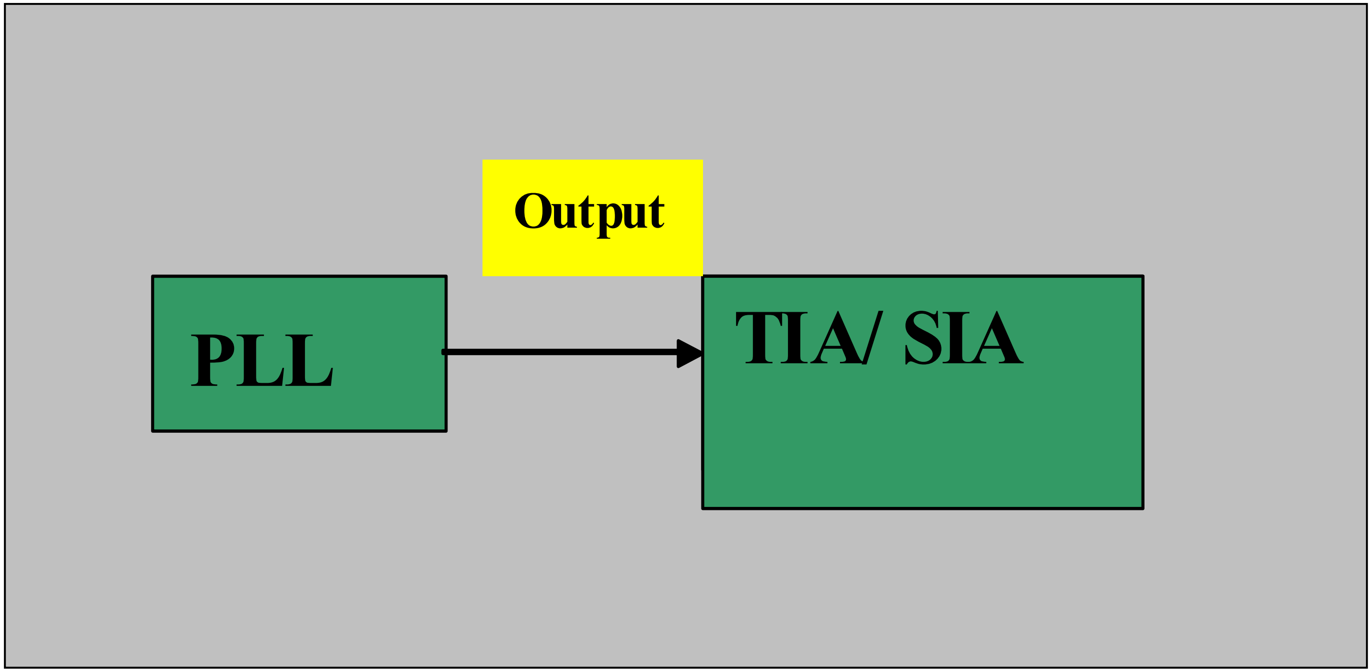
Limitations for Traditional Methods

- Requires a modulation & signal source and subject to modulation frequency limitation
- Not suitable embedded PLLs
- Cannot accurately measure PLL transfer function above 20 MHz
- No separation of from transfer function
- It is slow
- Only gives the magnitude part

New PLL Transfer Function Measurement

No stimulus is required, fast !



Advantages of the New Method

- No modulation & signal source is required
- Well suited for embedded PLLs
- Measure transfer function from ~ Hz to Nyquist
- It is fast (a complete PLL measurement and analysis takes ~ seconds)
- Separate noise from transfer function
- A complete, plug&play, fast solution
- Complete s-domain transfer function, prediction/simulation enabling

Conclusions and Remarks

- Variance/autocorrelation function based PLL analysis method and 2^{nd} order results are reviewed
- Third order PLL transfer function is derived and its performance is studied.
- A frequency domain based approach is proposed to achieve simultaneously measuring and analyzing PLL transfer function.
- The proposed method is scalable in terms of PLL order and thus backward compatible to the existing 2^{nd} order case.
- The method warrants a great promising and generic method for analyzing 3^{rd} or Nth order PLL jitter and transfer function with a great accuracy and throughput