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Statistical and System Transfer Function Based Method For Jitter and Noise In Communication Design and Test

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Abstract

As the communication speed/data rate approaches 1 Gb/s and beyond, timing jitter and amplitude noise become the major limiting factors for system performance. Traditional methods used in simulating, analyzing, modeling, and quantifying jitter and noise in terms of peak-to-peak and/or rms becomes no longer accurate and sufficient. As such, new methods with better accuracy and comprehension are called for. In this paper, we will first illustrate shortfalls of using simple parametric based method such as peak-to-peak or rms in quantifying jitter and noise. We then will discuss new jitter and noise modeling and analysis methods for both design and test based on statistical signal theory invoking probability density function (pdf) and cumulative distribution function (cdf) and the corresponding component distributions of deterministic and random to quantify jitter, noise, and Bit Error Rate (BER) for communication systems. Thirdly, we will introduce jitter and noise transfer functions and their important roles-played in estimating relevant jitter, noise, and BER in the system. Fourthly, we will introduce and illustrate how those methods can be used in designing and testing > 1 Gb/s communication systems, such as Fibre Channel (FC), Giga Bit Ethernet (GBE), and PCI Express.

Author Biography

Dr. Mike Li is currently the Chief Technology Officer (CTO) with Wavecrest. Dr. Li pioneered jitter separation method (Tailfit) and DJ, RJ, and TJ concept and theory formation. He has involved in setting and contributed to standards for jitter, noise, and signal integrity for leading serial data communications, such as Fibre Channel, Gigabit Ethernet, Serial ATA, and PCI Express. Currently he is Co-Chairman for PCI Express jitter standard committee. Dr. Li has more than 10 years experiences in high-speed related measurement instrumentation, testing, and analysis/modeling algorithms/tools, with applications in IC, microprocessor, clock, serial data communications for both electrical and optical, and wireless communication. Prior joining Wavecrest, Dr. Li had worked in both industry and academic institution. Dr. Li is experienced in measurement system and Automated Test Equipment (ATE) architectures, hardware, software, performance, and accuracy. He has a BS in physics from University of Science and Technology of China, a MSE in electrical engineering and a Ph.D. in physics from University of Alabama in Huntsville. He did his Post Dr. at University of California, Berkeley and worked there as a research scientist on high-energy astrophysics before he joined industry. Dr Li has published more than 40 papers in refereed technical journals, holds one patent and has four patents pending.

Mr. Wilstrup is a corporate consultant at Wavecrest. His present interests are SI simulation and analysis, signal and noise analysis methods and analog circuits. He has been an active member and major contributor leading serial data communication standards, such as Fibre Channel, Gigabit Ethernet, Serial ATA, and PCI Express. He invented autocorrelation/power-spectrum density based jitter separation method. He has published more than 10 papers in technical journals. He holds eight patents and has eight patents pending in the instrumentation area. He studied mathematics and physics at the University of Minnesota.

1. Introduction

The trend of communication devices and systems is that their speed and data rate keep increasing, and their prices for the same data rate keep going down. One of the key metrics in quantifying the performance of a communication system is BER. The major contributors to BER are timing jitter and amplitude noise on the digital bits of 0s and 1s. The timing jitter is typically defined as the time deviation from the ideal time location, while amplitude noise is the amplitude deviation from the ideal amplitude level. To maintain a same level of BER when the data rate increases, the jitter and noise need to be reduced accordingly so that the ratios of the jitter to the bit Unit Interval (UI, time period for a single bit) and noise to ideal amplitude are kept unchanged or even reduced. The levels of timing jitter and amplitude noise that matter for BER performance are in the orders of ps and mv respectively as the communication data rate approaching 1Gb/s and beyond. As such, comprehensive methods for modeling, analyzing, and testing jitter and noise for > 1 Gb/s communication systems are necessary in order to make those devices and systems with high performance and low cost.

Jitter and noise can have various sources. Commonly encountered are random noises such as thermal and flicker noises; bandwidth limited medium and device that produce duty cycle distortion (DCD) and inter-symbol interference (ISI); unintended/intended modulation that causes periodic jitter or noise; unmatched interface that causes waveform distortion; crosstalk and electrical and magnetic interference (EMI) that cause bounded-uncorrelated jitter and noise. Depending on the exact root cause, different jitter or noise sources will have different characteristics. The single most important aspect for jitter and noise is that deviation is *not* an instantaneous single sample event, but rather is a long-term, many sample events process. As such, a statistical treatment for jitter and noise is required, particularly at > 1 Gb/s rate.

Historically, jitter and noise are studied by using simplified statistical parameters such as peak-to-peak and rms values. In the presence of random jitter and noise, the peak-to-peak is sample size dependent and cannot be a repeatable and reliable statistical measure, while rms measure can also be misleading when there are non-random components in the distribution[1]. As the goal becomes to completely grasp the jitter and noise processes, and to quantify the overall distribution and its associated components and root causes, the simple parameter based approach to jitter and noise becomes insufficient and invalid.

While a complete statistical treatment of jitter is necessary, another important aspect of jitter and noise needs to be considered is the jitter transfer function for the communication system. To accurately estimate BER for a communication system, only the relevant jitter and noise needs to be considered. *Underestimated* or *overestimated* jitter and noise can cause either *liability* or *yield* problems respectively, and the accuracy is the key. An accurate estimation of BER and relevant jitter and noise will not be possible unless the jitter/noise transfer function is incorporated in the estimation process.

It becomes clear that new simulation, modeling, and analyzing methods are needed for jitter and noise in the multiple 1 Gb/s era. In section 2, we will discuss the critical issues when using peak-to-peak parameter in estimating total jitter, particularly in the presence of random component. In section 3, we will discuss statistical distribution function based methods to treat jitter and noise and show their advantages and gains over simple parameter based method in terms of accuracy and completeness. In section 4, we will discuss the jitter transfer function of a communication system and show how to use it in conjunction with the jitter distribution function to accurately and completely determine the system

BER, total jitter and noise, and various jitter and noise components. In section 5, we will discuss how will the statistical and system transfer function based jitter, noise, and BER analysis and measurement method be used in practical communication architectures such as Fibre Channel, Giga bit Ethernet, and PCI Express. We summarize and conclude in section 6.

2. Peak-to-peak error and its dependence to sample size

We have shown that the peak-to-peak for a jitter histogram is a function of sample size in the presence of random jitter[1]. The nature question to ask will be: how well is the peak-to-peak repeatability and accuracy in the presence of random jitter, particularly in regard to total jitter (TJ) at BER of 10^{-12}? A Monte Carlo simulation will be a powerful tool to address this issue.

In our simulation, the true total jitter is 1 at BER = 10^{-12}, simulated peak-to-peak, an equivalent way to measure the total jitter at a given sample size, is normalized to unit 1. The total jitter model is a "double delta" deterministic jitter (DJ) convolved with a single Gaussian random jitter (RJ). For each combination of DJ peak-to-peak and RJ sigma, 200 (M in the simulation) peak-to-peak "measurements" were performed. The mean of these 200 "measurements" is calculated, to improve the accuracy of the peak-to-peak estimate. Also, for each measurement, the sample size N, is defined. N is the number of active clock transitions or the number of data transitions. The simulation results are applicable to either clock jitter or data jitter measurement.

Table 1: Mean Peak-to-Peak Jitter with Measurement Number of M=200

TJ cases		N			
$\Phi_{dj\_pk\text{-}to\text{-}pk}$	$\Phi_{rj\ sigma}$	10^3	10^4	10^5	10^6
0.0	0.0711	0.458	0.544	0.623	0.693
0.2	0.0577	0.547	0.622	0.688	0.746
0.4	0.0432	0.660	0.717	0.766	0.810
0.6	0.0288	0.773	0.811	0.844	0.873
0.8	0.0144	0.887	0.906	0.922	0.937
1.0	0.0000	1.000	1.000	1.000	1.000

For all cases except: DJ=1.0 peak-to-peak and RJ sigma is 0.0, the total jitter has a width of 1 at a probability of 10^{-12}.

For DJ=0.0, the RJ sigma is 0.0711; the reciprocal of this is about 14.For total jitter having only Gaussian RJ, the width of the total jitter at a probability of 10^{-12} is about 14 sigma.

Some trends in the Table 1:

a.) When RJ sigma>0, the peak-to-peak measurement always underestimates the total jitter (unit 1 in this example), at a probability of 10^{-12}.

b.) The amount of underestimation decreases as the sample size, N, increases.

c.) The quantity of underestimation is a function of the relative amounts of DJ and RJ. For less amounts of RJ, the underestimation is less severe.

Also, if $N>>10^{12}$, the peak to peak measurements for RJ sigma > 0 would overestimate the total jitter at a probability of 10^{-12}.When $N<<1/10^{-12}$, all jitter methods *grossly* under sample and it doesn't matter how the hardware that performs the raw jitter measurements is designed and it doesn't matter how these raw jitter measurements are processed! The answer to this problem is to extrapolate the measurements using a total jitter model that will be discussed in next section.

What does the peak-to-peak distribution look like, figure 1 gives the answer

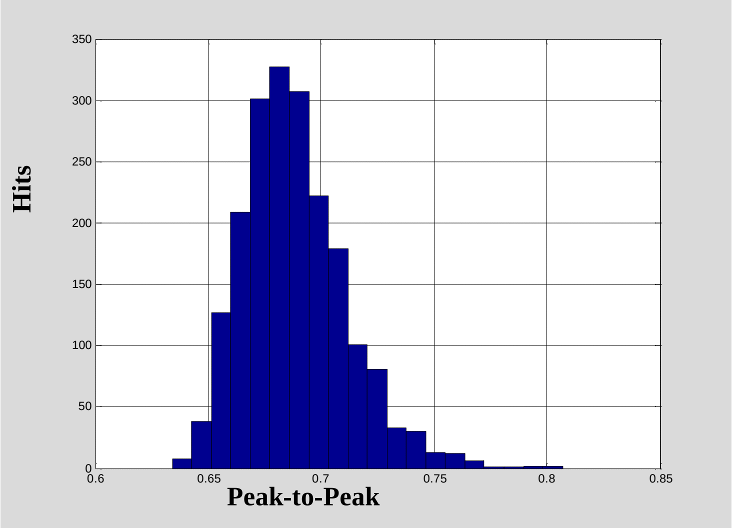

Figure 1: Distribution of a peak-to-peak, with 2000 measurements, and 100,000 samples for each measurement. The parameters for jitter model are: DJ = 0.2, RJ sigma = 0.0577 .

The distribution is non-Gaussian and un-symmetrical; therefore do not expect average will reduce the random error.

One way to estimate the error for a distribution is to calculate the ratio of standard deviation to its mean from M measurements, we termed here as “normalized standard error”. The following table shows the results:

Table 2: Normalized Standard Error of the Peak-to-Peak Jitter
with Measurement Number of M=200

TJ cases		N			
$\Phi_{dj\_pk\_to\_pk}$	$\Phi_{rj\ sigma}$	10^3	10^4	10^5	10^6
0.0	0.0711	0.073	0.053	0.044	0.036
0.2	0.0577	0.052	0.040	0.032	0.029
0.4	0.0432	0.032	0.026	0.022	0.020
0.6	0.0288	0.018	0.015	0.013	0.012
0.8	0.0144	0.008	0.007	0.006	0.006
1.0	0.0000	0.000	0.000	0.000	0.000

The normalized standard error is a measure of the repeatability of the "measurements". The repeatability adds to the systematic errors in measurement and increases the total measurement error.

As the RJ sigma gets larger and/or N gets smaller, the repeatability becomes worse.

If the distribution of the measurement was Gaussian, the error of the measurement due to repeatability would be simple to calculate.

Anyway, for a rough estimate, multiply the normalized standard errors by 2 and convert to percent. Example, for DJ=0.2 and $N=10^6$, this is about 5.8 %, which is poor for measurement.

The central problem with peak-to-peak measurement is that only the extreme points of the data are used to estimate the variability of the data. This is *not* an efficient use of measurement data. In short, peak-to-peak does not warrant an accurate and repeatable measure for total jitter unless the sample size is close the reciprocal of the targeting BER, such as 10^{-12}.

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3. Statistical distribution functions for jitter and noise

3.1 Jitter pdf and cdf functions

Jitter and noise are statistical processes. The suitable and correct mathematical method to deal with a statistical jitter and noise processes is through using their pdfs of $P(\Delta t)$ and $P(\Delta v)$. For simplicity, we will only give detailed mathematical discussion on timing jitter pdf $P(\Delta t)$ since the methods developed for timing jitter pdf can also be applied to amplitude pdf $P(\Delta v)$ in most cases.

A timing jitter pdf $P_t(\Delta t)$ can be further deconvolved into two component pdfs of deterministic $P_d(\Delta t)$ and random $P_r(\Delta t)$. Deterministic pdf is defined as bounded, while random pdf is defined as Gaussian and is unbounded. The convolution is shown mathematically in the following equation (1):

$$\begin{aligned} P_t(\Delta t) &= P_d(\Delta t) * P_r(\Delta t) \\ &= \int_{-\infty}^{+\infty} P_d(t) P_r(\Delta t - t)\, dt \end{aligned} \tag{1}$$

Once the deterministic and random pdfs are determined through deconvolution and that is an inverse operation of equation (1), all the appropriate statistical parameters, such as mean, rms, peak-to-peak, are readily calculated for each pdf.

When the RJ distribution is Gaussaian, its characteristics are completely described by its mean and rms. Conversely, once those two parameters are known, Gaussian random pdf is uniquely determined. A blank peak-to-peak value is *not* meaningful since Gaussian is unbounded. For an unbounded pdf, a peak-to-peak should always be mentioned with its probability density level. For a bounded pdf, peak-to-peak only gives *a partial* picture since from that, no unique pdf can be established. To avoid the “loss of information” problem, deterministic component should be always quantified with its pdf, rather than a degenerated single peak-to-peak number.

With the complete $P_t(\Delta t)$ for the timing jitter, the BER can be estimated. Figure 2 gives an illustration on the relationship between jitter pdf and BER in the context of an eye-diagram. 0 and 1 denote the first and second ideal zero-crossing time locations for the bit cell. If an edge transition happens at the right side of the sampling time ts for the first zero crossing jitter pdf, then an bit error occurs; similarly, if an edge transition happens at the left side of the sample time ts for the second zero-crossing jitter pdf, an bit error also occurs.

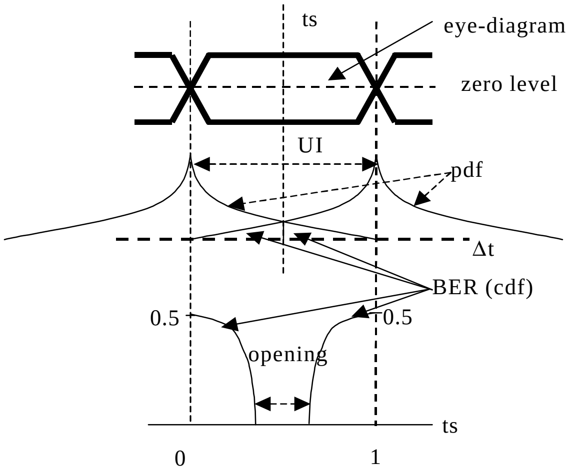

Figure 2. Illustration of the relationship between eye-diagram, jitter pdf, and BER (cdf) function.

The BER function is the sum of those two “tail” jitter pdf areas. Since a cdf function is the integral of a pdf function, as such BER function is essentially the jitter cdf function, as shown in equation (2)

$$BER(t_s) = cdf(t_s) = \frac{1}{2}\left[\int_{t_s}^{\infty} P_{t0}(\Delta t)d(\Delta t) + \int_{-\infty}^{t_s} P_{t1}(\Delta t)d(\Delta t)\right] \qquad (2)$$

For most > 1 Gb/s communication systems, a total jitter (TJ= UI- opening) at BER = 10^{-12} or better is required. This implies that an accurate determination of BER function down to BER =10^{-12} or lower is necessary.

3.2 Deconvolution of jitter pdf

The first driving force for jitter and noise deconvolution is that determining BER or cdf down to 10^{-12} is a non-trivial task from perspectives of both simulation and test. To illustrate the point, let’s assume a Gaussian jitter pdf. At BER = 10^{-12}, the sample size needed to establish the corresponding BER/cdf needs to be > 10^{12}. That means at least 10^{12} edge transition samples need to be obtained. The simulation time, as well as test time to acquire this much sample will be very long, typically in hours or longer. This implies that alternative methods capable of determining BER/cdf function down to 10^{-12} with good throughput and accurate is needed. The second driving force is for better understanding, analysis, diagnosis, and debug purposes since by knowing the jitter components and corresponding pdfs, identifying design flaw, source of error, and failure mechanism can be made quickly and accurately.

3.2.1 Tailfit method

The concept of Tailfit[2] is relatively straightforward. Since the deterministic pdf is bounded, so beyond certain jitter range, all the pdf will be solely corresponding to random jitter process. Those regions where Gaussian process dominant are typically in the tail parts of the total jitter pdf. When an analytical Gaussian form is used to match those tail region pdfs through least-square fit or other optimization procedures, all the parameters defining a Gaussian distribution such as mean and rms will be determined. Once the Gaussian pdf is determined, the deterministic pdf and associated parameters can be further determined through a secondary deconvolution[3]. Tailfit method can be applied to both timing jitter and amplitude pdfs. The Tailfit method is shown schematically in figure 3.

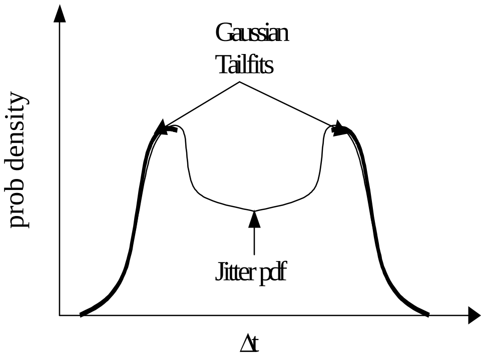

Figure 3. Schematic of Tailfit to a total jitter pdf

3.2.2 Variance method

Tailfit can be applied to single pdf, or multiple pdfs, or its secondary function such as cdfs. If those pdfs are obtained over a span of edge transitions, then both deterministic jitter and random jitter time series can be determined by applying Tailfit to all pdfs and the related spectrum analysis can be conducted through Fourier Transformation (FT) or Fast Fourier Transformation (FFT). Another way to do the spectrum analysis is through the calculation of mean and variance/autocorrelation functions of jitter time series[4]. The mean of time deviations from ideal gives the DCD&ISI estimation, while the variance function in frequency domain gives the power spectrum density (psd) for periodic and random jitter. The mathematical process in obtaining jitter variance function from the jitter time series is shown in the following equation (3)

$$VAR(f) = FFT\,(c - 2R_{xx}(\Delta t(n))) \tag{3}$$

where c is a constant and R_{xx} is the autocorrelation function. Through appropriate frequency band normalization, frequency domain variance function VAR(f) becomes the psd function. A typical jitter psd function having both periodic and random jitter looks like the following figure 4.

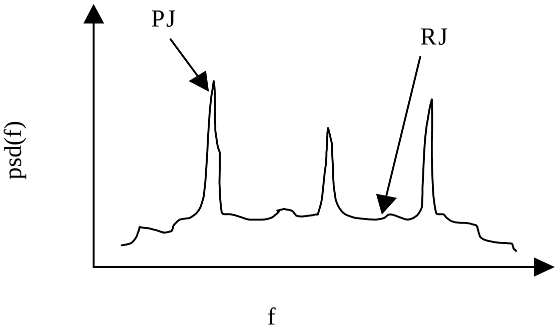

Figure 4. Schematic of jitter psd. Spikes are PJs and the spectral envelope with PJs removed is RJ psd.

Since any PJs will be shown as spectral lines in the psd(f) function, and a "sliding window" technique can be used to identify the PJ magnitudes and frequency locations. Then all the PJ spectral lines are removed from the psd(f) record. The residues are summed over certain frequency band and the square root of the sum gives the RJ rms value over that frequency band.

4. Jitter frequency response function

At data rates > 1 Gb/s, most of the communication architectures are serial where the clock is embedded in its transmitting data bit stream. At the receiver side, this clock needs to be recovered through a clock recovery device where PLL circuits are commonly used. It is well-known that a PLL has certain frequency response characteristics. Therefore, when a receiver uses the recovered clock to time/retime the received data, the jitter "seen" by the receiver will follow certain frequency response function as well. Figure 5 shows a receiver and its clock and data recovery subsystems.

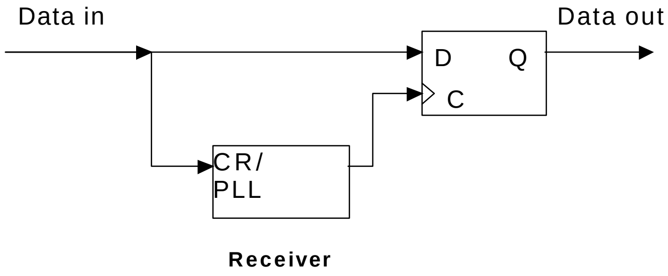

Figure 5. A schematic block diagram for a serial receiver with clock and data recovery.

A PLL typically has a low-pass frequency response function $H_L(s)$ (s is a complex frequency) as shown in the following figure 6.

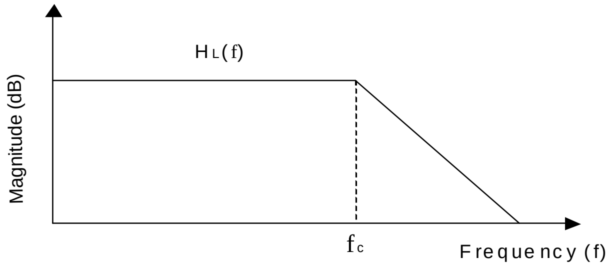

Figure 6. A typical PLL magnitude frequency response

Any good simulation or test methodology should emulate the actual device behavior. In the case of receiver jitter determination, the model setup for both design and test should be such that *it determines the jitter as what a receiver sees*. A receiver sees jitter on the data from its recovered clock; therefore it is a difference function from clock to data as shown in the following figure 7.

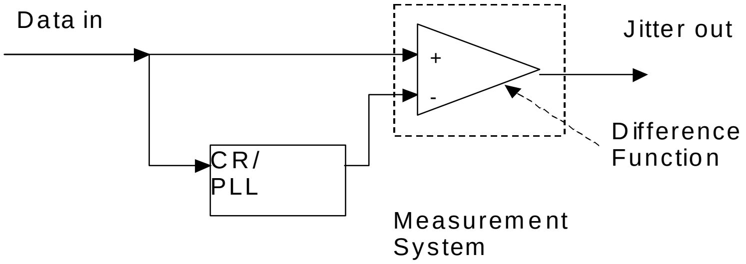

Figure 7. A jitter model emulates jitter as seen by a serial data receiver. Note that the data latch function of "D" flip-flop in figure 5 is replaced by the difference function to emulate the receiver jitter behavior.

Because the clock recovery (or PLL) device has a low-pass transfer function $H_L(s)$, the jitter output will have a high-pass transfer function of $H_H(s)$ as shown in the following figure 8 since $H_L(s) + H_H(s) = 1$.

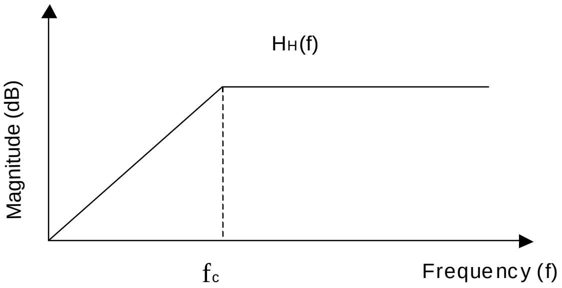

Figure 8. Jitter transfer function magnitude response as seen by a serial receiver or as measured by a difference function.

The high-pass jitter transfer function $H_H(s)$ shown in figure 8 suggests that a receiver is able to track or reject more low-frequency jitter at $f < fc$ than at higher frequencies of $f > fc$. This implies that a receiver can tolerant more low-frequency jitter than high-frequency jitter. The jitter tolerance function is the "mirror" function of the jitter transfer function around the unit gain.

5. Systematical application of jitter pdf, cdf, and transfer functions

To determine the relevant jitter for a communication system, either from design or test perspectives, the receiver jitter response needs to be incorporated. The timing jitter pdf is referenced to a clock that has a certain frequency response. In the case when a PLL is used to recover the clock from the data stream, this reference clock is the PLL recovered clock and the jitter transfer function is complementary to the PLL transfer function. Once the jitter pdf folded in with the jitter transfer function is determined, then the cdf (or BER function), deterministic and random pdfs, as well as appropriate statistical parameters can be determined by using the methods introduced in section 2.

For Fibre Channel (FC)[5], and Giga Bit Ethernet (GBE) [6] jitter compliance test, a first-order "golden" PLL is assumed for the standards. As such transfer function is an idealized first order high-pass function with a corner frequency set to be $f_c = f_d/1667$, where f_d is the data rate. The overall shape of the high-pass function is similar to what is shown in figure 8, with the slope below the corner frequency f_c being 20 dB/decade. Apparently, this is a simplified approximation to an actual PLL used in a practical receiver where most PLLs used are second order or higher for better performance. If the PLL for the clock recovery is second order, then the high-pass function of the jitter transfer will be 40 dB/decade at frequencies below fc. This means that the receiver can track/reject more lower frequency jitter below fc in comparison with a receiver with a first-order PLL as its clock recovery. Consequently, it can also tolerate more lower-frequency jitter than a receiver that has a first-order PLL as its clock recovery.

For PCI Express, the clock recovery can have different implementations, such as phase interpolation, PLL, or over sampling[7]. As a result, there will be different jitter transfer functions associated with each implementation. The current specification gives a "250 UI" time record to estimate the jitter in the corresponding frequency range. However, such a definition does not lead to a clear and precise frequency-domain jitter transfer function needed to estimate jitter accurately for different implementations. Work is in progress within PCI Express standard committee to address this frequency response issue.

6. Summary and conclusion

We have pointed out that both jitter and noise become the major performance limiting factors for a communication system as its data rate reaching 1 Gb/s or beyond. We have demonstrated that simple parametric based methods such as those using only peak-to-peak or rms have become insufficient and somewhat invalid in simulating, modeling, and testing communication systems at > 1 Gb/s rates and new comprehensive methods with better accuracy and coverage are needed. We further illustrated that statistical based methods invoking distribution function such as pdf and cdf are capable of quantifying jitter and noise correctly and completely. We further illustrated why deconvolving jitter and noise pdfs into distinct sub-pdfs such as deterministic and random is necessary to accurately identify the root sources and quantify them with appropriate statistical parameters. We introduced two methods to conduct the deconvolution and spectrum analysis, one is Tailfit based, and another is variance function based.

We then view the jitter and noise from the system perspective. We first reviewed the clock and data recovery architecture in a serial data communication system. We then demonstrated that the frequency response function of the clock recovery plays an important role in tracking out low frequency jitter at the receiver. We have pointed out that in order to emulate a receiver behavior, a difference function between recovered clock and data is required. We have shown that the jitter frequency response is complementary to the frequency response of the clock recovery. We also have shown that the jitter transfer function has a high-pass characteristic due to the fact that most clock recover unit has a low-pass characteristic. We pointed out that the "mirror" function of the jitter transfer around unit gain gives the jitter tolerance function for the receiver. The exact jitter transfer function depends on how clock and data recovery is implemented in a receiver.

Having discussed jitter and noise from perspectives of both statistical and system transfer function, we demonstrated how to apply those methods in simulating, modeling, and test jitter and noise for > 1 Gb/s communication systems, such as Fibre Channel, Giga Bit Ethernet, and PCI Express.

In summary, any valid methods for simulating, modeling and testing jitter and noise for a multiple Gb/s communication system must grasp the statistical and system transfer function characteristics in order to achieve accurate, complete, and repeatable estimations and analyses. We have shown a few methods that can meet those requirements.

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