Relationship Between Eye Diagram and Bathtub Curves

Technical Bulletin #13

Introduction

Two diagnostic tools commonly used in signal integrity analysis are eye diagrams and bathtub curves (Figure 1). Eye diagrams (Figure 1a) are created with sampling oscilloscopes and consist of multiple traces of data bits triggered by a bit clock. The traces are superimposed in persistence mode showing the envelope of amplitude and timing fluctuations. The region in the center that is devoid of any traces typically resembles an eye, thus the descriptive name "eye diagram." The eye diagram can only qualitatively show the range of amplitude and timing deviations associated with the data. An eye diagram with a large eye opening is indicative of a data stream that has very little amplitude and timing noise. An eye diagram with a small opening (eye closure) indicates that the data is very noisy.
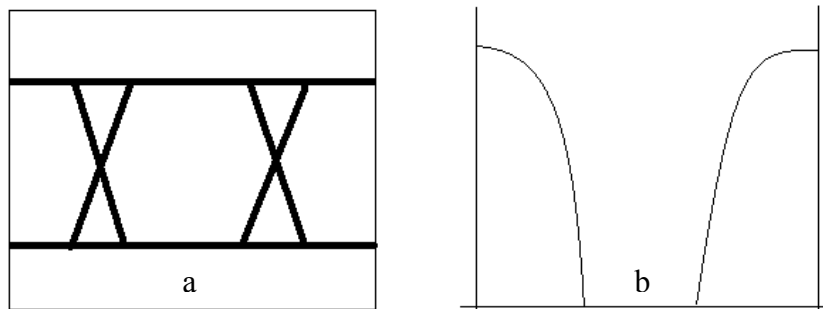

Figure 1 - Examples of eye diagram (a) and bathtub curve (b).

Bathtub curves (also referred to as "BERT scans") are usually created with bit error ratio testers (BERT). A BERT generates data to pass through a device under test (DUT) and then measures the transmitted data and compares for errors, thus determining the bit error ratio (BER). As the measurement location is swept across a unit interval (UI), a plot of BER as a function of the UI is constructed. This plot typically resembles a cross section of a bathtub, thus the name "bathtub curve" (Figure 1b).

Relationship Between Eye Diagram and Bathtub Curve

A histogram of the timing location of data transitions passing a voltage threshold can be compiled. Such a histogram can be acquired with an equivalent-time, sampling oscilloscope. However, the acquisition rate is generally slow and requires a very long time to achieve a high statistical count. The preferred method for histogram acquisition is by using a time interval analyzer (TIA, such as the *WAVECREST* SIA-3000).

In a short time (several seconds), histogram measurements will capture the most probable timing locations of data transitions. This histogram is also known as a jitter histogram. If the histogram is rescaled such that the integral is unity, it becomes a probability density function (PDF) of jitter (Figure 2b). It is understood that the total jitter PDF is a convolution integral of bounded deterministic jitter (DJ) and unbounded Gaussian random jitter (RJ). Because DJ is finite and bounded, the extremes of the jitter PDF must consist only of RJ Gaussian "tails." Thus, the rms standard deviation of the Gaussian can be found from least squares fitting of these tail regions (TailFit™). Extrapolation of the fit allows the determination of the probability density of data transition locations that are very rare and are not captured in the limited measurement time.

Once the complete jitter PDF is determined from the jitter histogram with TailFit™ applied, the corresponding bathtub curves can be easily found. Recall that the range of the bathtub curve is a dimensionless number BER, which is the probability of bit errors. The indefinite integral of the complete jitter PDF is the probability of bit errors due to timing. Thus, the bathtub curve of timing errors is simply an integral, or the cumulative density function (CDF), of the jitter PDF (Figure 2c). For the probability of eye closure from the left side, the jitter PDF is integrated from right to left.

Eq. 1
$$BER_{right}(t) = \int_{-\infty}^{t} PDF(t')dt'$$

Similarly, for the right side, the jitter PDF is integrated from left to right.

Eq. 2
$$BER_{left}(t) = \int_{t}^{\infty} PDF(t')dt'$$

Because TailFit™ can predict the probability of extremely rare bit errors, the bathtub curves for BER as low as 10^{-16} can easily be found. The measurement and analysis time by this method is a matter of seconds. To generate a bathtub curve down to 10^{-12} BER with a BERT generally requires many hours.
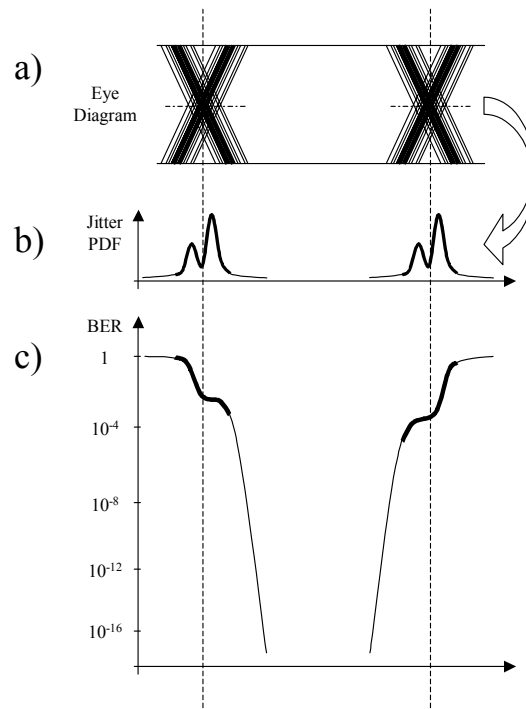

Figure 2 - Illustration of relationship between eye diagram, jitter PDF, and bathtub curve. a) Eye diagram indicating data transition threshold. b) Jitter PDF (thick line) with TailFit™ extrapolation (thin line). c) Bathtub curves found from jitter PDF (thick line) and TailFit™ extrapolation (thin line).

Conclusion

Both eye diagrams and bathtub curves are methods of signal integrity analysis and are related to the jitter PDF. However, eye diagrams generally suffer from low sample sizes and trigger jitter. Thus, an eye diagram is only a qualitative analysis. Bathtub curves generated by a BERT are generally considered to be very accurate and precise. However, testing times often take many hours.

A TIA (e.g., *WAVECREST'S* SIA-3000) can measure a jitter PDF quickly and accurately. Jitter events that have an extremely low probability will not be captured by the TIA. But with *WAVECREST'S* TailFit™ algorithm, these "outliers" can be predicted based on statistical modeling. With a complete jitter PDF, bathtub curves with BER down to 10^{-16} can be generated accurately.