# Characterizing Jitter Histograms

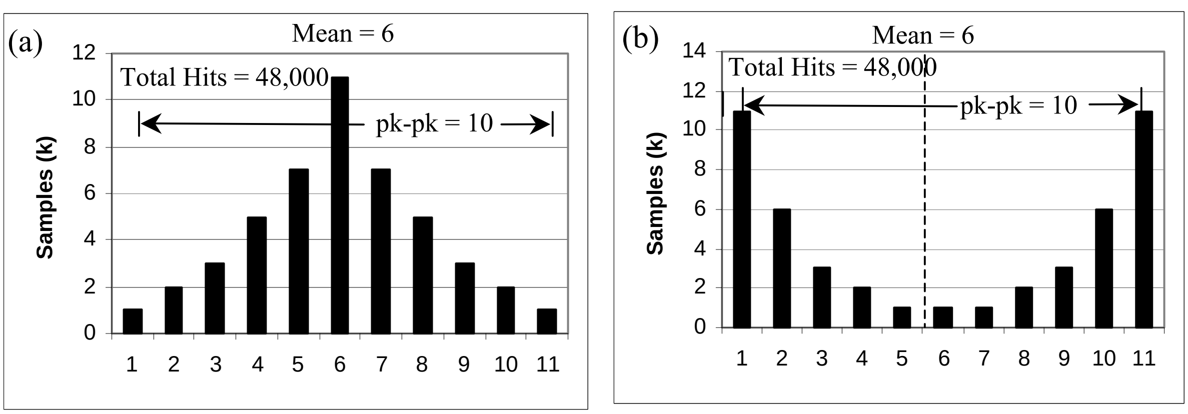
Technical Bulletin #12

## Introduction

Clocks, oscillators and Phase Lock Loops (PLLs) are a few of the most basic, yet vital, building blocks in today's high-speed communication systems. Because these components are responsible for critical timing applications in systems, it is important to accurately characterize and quantify their jitter in order to determine system performance and reliability. Traditionally, it was believed that the component performance and/or reliability could be determined with a simple peak-to-peak value. This document will discuss the drawbacks of quantifying histograms with peak-to-peak measurements and will recommend other, more thorough characterization methods that reflect true device performance thereby enabling engineers to accurately make pass/fail decisions.

## Gaussian Distributions

Intuitively, one would expect that two different devices with the same peak-to-peak jitter values would have the same performance. However, that conclusion cannot be made without further analysis. For example, figures 1(a) and (b) show two different distributions with the same number of total hits, mean and pk-pk values. Clearly distribution (b) has more variability since there are fewer hits near the mean. The biggest drawback with quantifying a distribution with a pk-pk value is that it is based on only two points and ignores the rest of the distribution.[1] Most important, however, is quantifying a distribution based on only two peak points can result in poor measurement repeatability.




Figures 1(a) and 1(b) show two bar charts with the same number of hits and a pk-pk of 10 (11-1), but with dramatically different distributions.

One way to measure variability in a distribution is to calculate the standard deviation or $\sigma$ (sigma). An important property of the standard deviation is that the calculation involves every point in the distribution and the $\sigma$ stabilizes quickly as a function of sample size. Figure 2(a) illustrates that the $\sigma$ is very stable as a function of samples in the distribution, however the pk-pk values increase with sample size. When quantifying the performance of a device it is important to use parameters that do not change as a function of a variable, such as sample size. In other words, a distribution with a particular pk-pk value has meaning only if it is given with sample size and test conditions. However, the accuracy of the pk-pk measurement is low. Figure 2(b) shows standard error for both pk-pk and standard deviation as a function of sample size. The standard error quantifies the uncertainty in a measurement. The standard error of the pk-pk measurement is larger than the $\sigma$ for all sample sizes. Therefore, the standard deviation is more repeatable for describing a distribution because the value quickly stabilizes and has less error compared to pk-pk measurements.

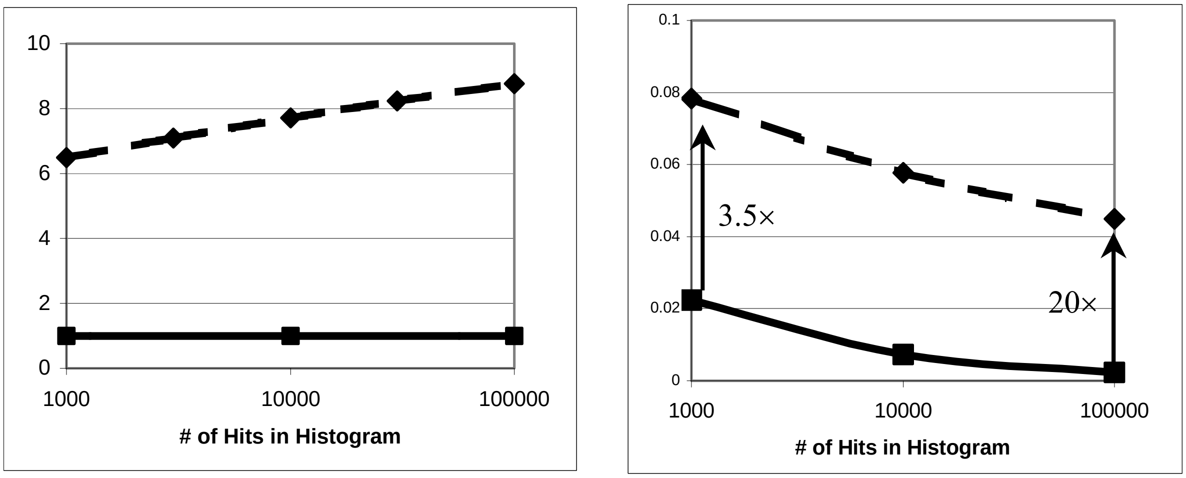



Figure 2(a) plots the σ (solid line) and pk-pk (dashed line) values as a function of sample size for a Gaussian variable. The figure shows that the pk-pk measurements increase with sample size, whereas the σ is nearly unchanged. Figure 2(b) shows the standard error for the σ (solid line) and pk-pk (dashed line) versus sample size. For all sample sizes the pk-pk measurements have a larger standard error compared to the σ. In addition, the accuracy of the pk-pk measurements relative to the σ gets worse with increasing sample size.

For Gaussian distributions the standard deviation provides additional insight into the distribution's characteristics and can be used to predict pk-pk jitter as a function of probability level. The σ is often times referred to as the "width parameter." Large σ values imply a wide "bell shaped" distribution and a small σ value implies a narrow "bell shaped" distribution. Random jitter (RJ), seen in all clocks, oscillators and PLLs, is characterized by a Gaussian distribution and is assumed to be unbounded. Knowing the standard deviation of a Gaussian distribution is useful because it can be used to calculate the width of the distribution for a given probability level. In many data communication standards, it is common to express pk-pk jitter at a given BER.[2]. For example, a BER of $1.3 \times 10^{-3}$ would be a pk-pk range of $6 \times \sigma$. This means that the pk-pk range specified by $6 \times \sigma$ would contain all of the measurements except an amount represented by multiplying the total number of measurements by 0.0013.

## Non-Gaussian Distributions - Real life distributions

It is very common to observe jitter histograms of clocks, oscillators and PLL's that are not ideal Gaussian distributions that contain a mixture of Gaussian and non-Gaussian histograms. Quantifying a non-Gaussian distribution with the σ and determining pk-pk jitter as a function of probability level is not valid so other methods must be employed. Figure 3(a) shows three different histograms with the same σ. Figure 3(b) shows BER or P(x) versus σ for the three histograms. Figure 3(a) illustrates that even though the three histograms have the same σ, they have dramatically different pk-pk ranges for a given BER. For example, if these were jitter histograms, the Gaussian distribution (blue) at $10^{-12}$ BER would have a pk-pk jitter of $14 \times \sigma$ whereas distribution (green) would have a pk-pk jitter of $11.5 \times \sigma$ with (red) being only $9.2 \times \sigma$. Both the green and red histograms contain a mixture of deterministic and random components.[2] One method of separating the two components for any shaped histogram is accomplished by fitting Gaussian tails to the left and right side of the histogram, commonly called the TailFit™ method.[3] This method quantifies the random jitter (RJ) component and the difference between the means of the two Gaussian curves is the deterministic jitter (DJ). An example is shown in Figure 4 for a tri-modal histogram where Gaussian tails are fit to the left- and right-most sides of the histogram. Quantifying RJ and DJ allows one to accurately calculate the pk-pk jitter as a function of BER. Furthermore, knowing the RJ and DJ components provides additional insight into system performance. The magnitude of these values, relative to a specification, will enable designers and engineers to use other diagnostic tools to locate the problem. For example, a large DJ component may suggest unwanted interference.

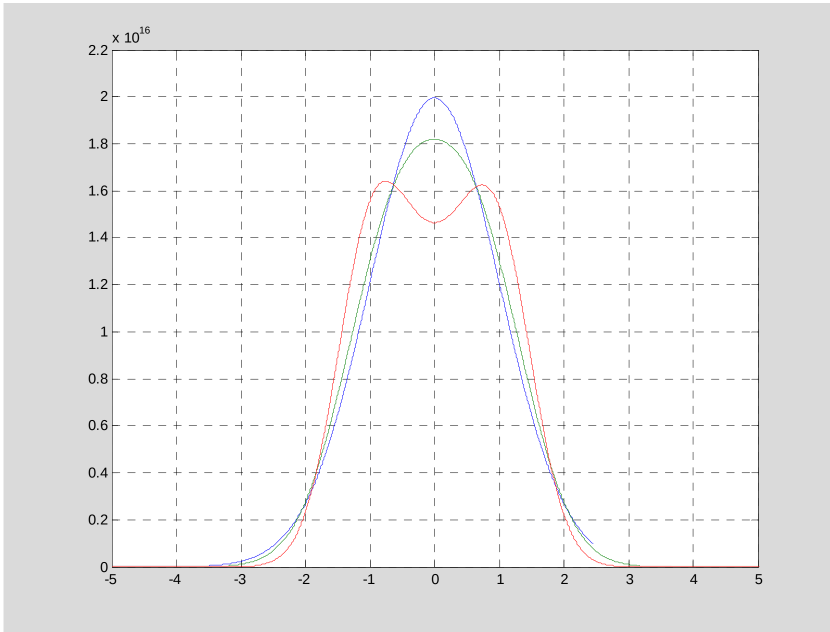
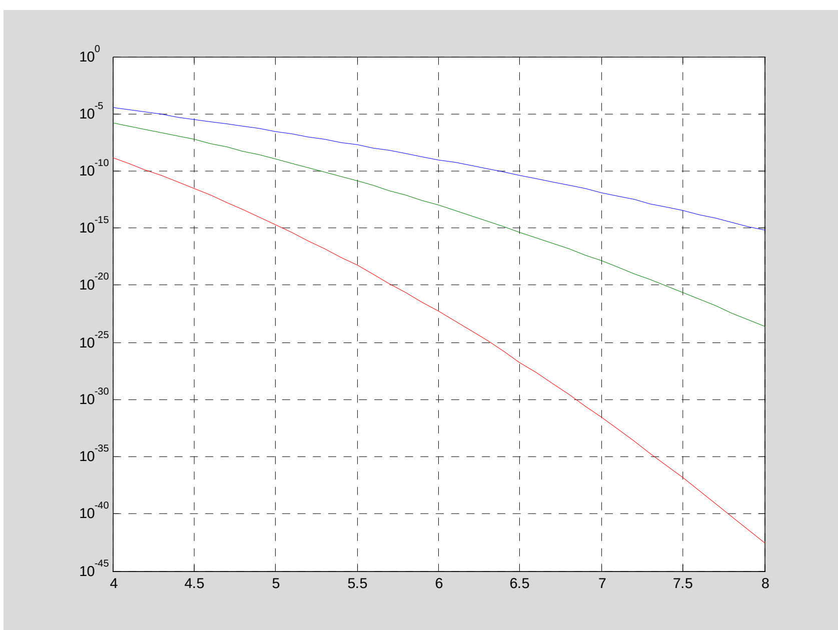



Figure 3(a) -Three different histograms with the same standard deviation. Gaussian distribution. (blue) Gaussian distribution with a small amount of PJ (green) and Gaussian distribution with significant PJ (red).




3(b) - Resultant curves showing BER as a function of standard deviation.

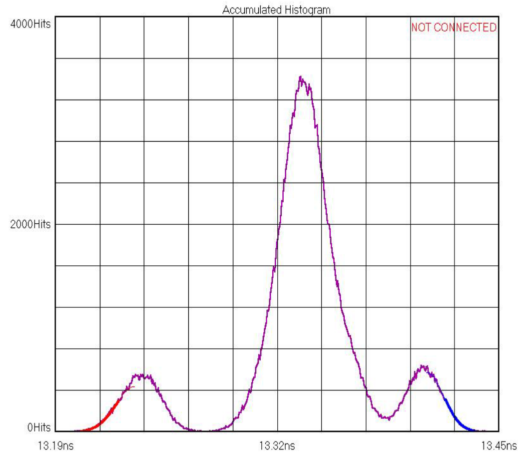



Figure 4 - A jitter histogram of period measurements that contains both RJ and DJ. To accurately quantify random jitter and deterministic jitter, tails have been fit with Gaussian distributions on the left (red) and right (blue).

## Conclusions

This document has reviewed a variety of ways to quantify jitter distributions. It was shown that a pk-pk number does not adequately quantify a distribution because it is inaccurate and its magnitude depends on sample size. Standard deviation provides a good metric of describing Gaussian distributions because it quickly converges to a stable value with increased sample. The magnitude of the standard deviation also provides a relative measure of a distributions width and pk-pk jitter can be determined as a function of probability level or BER. For non-Gaussian or "real life" distribution, it was shown that the random and deterministic components need to be quantified in order to correctly calculate jitter and relate it to system performance. The conclusions are summarized below in Table 1.

| Parameter | Benefits | Drawbacks |
|---|---|---|
| Pk-pk measurement of a distribution | • Provides a number | • Measurement must be stated with sample size and setup conditions.<br>• Measurement not repeatable |
| Standard deviation ($\sigma$) | • Measurement parameter is repeatable<br>• Can be used to calculate pk-pk jitter as a function of BER or probability level | • Useful only for Gaussian distributions |
| Quantifying random and deterministic components | • Can be used to calculate pk-pk jitter as a function of BER or probability level for any shape of histogram.<br>• The magnitude of the components provides diagnostic information | |

### References

[1] Freund, Rudolf J. and Wilson, William J., Statistical Methods, Rev ed. San Diego: Academic P, 1997.
[2] "Jitter Fundamentals." Wavecrest Corporation, 2000.
[3] Li, M. P., Wilstrup, J., Jessen, R. and Petrich, D., "A new method for jitter decomposition through its distribution tail fitting", ITC Proceeding, 1999.